2021/TDC (CBCS)/EVEN/SEM/
MTMSEC–601T/130

TDC (CBCS) Even Semester Exam.,
September—2021

MATHEMATICS

(6th Semester)

Course No. : MTMSEC–601T

(Analytical Geometry)

Full Marks : 50
Pass Marks : 20

Time : 3 hours

The figures in the margin indicate full marks for the questions

SECTION—A

Answer any *fifteen* of the following questions : $1\times15=15$

1. If the rectangular axes rotated through an angle θ without changing the origin, and (x, y); (x', y') are the coordinates of a point w.r.t. original and new system respectively, then write down the value of x in terms of x' and y'.

2. Write down the form of the equation $3x+4y=5$ due to change of origin to the point $(3, -2)$ only.

3. When is an expression said to be invariants in orthogonal transformations?

4. A second-degree homogeneous equation of the form $ax^2 + 2hxy + by^2 = 0$ represents a pair of straight lines through the origin. Write down the condition that the two lines are coincident.

5. What is the angle between the pair of lines given by the equation $ax^2 + 2hxy + by^2 = 0$?

6. What is the condition that the equation $ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$ represents two perpendicular straight lines?

7. Define orthogonal circles.

8. Write down the condition of orthogonality of the two circles $x^2 + y^2 + 2gx + 2fy + c = 0$ and $x^2 + y^2 + 2g'x + 2f'y + c = 0$.

9. Define radical axis of two circles.

10. Define coaxial circles.

11. Write down the equation of tangent to the circle $x^2 + y^2 = a^2$ at (x_1, y_1).

12. Write down the limiting points of the system of coaxial circles $x^2 + y^2 + 2gx + c = 0$.

13. Write down the length of the chord intercepted by the parabola $y^2 = 4ax$ on the straight line $y = mx + c$.

14. Write down the condition that the line $y = mx + c$ is a tangent to the parabola $y^2 = 4ax$.

15. Define auxiliary circle on an ellipse.

16. Define diameter of a conic.

17. Write down the parametric equation of hyperbola.

18. Write down the equation of tangent of the hyperbola $xy = c^2$ at the point $(ct, c/t)$.

19. What is the length of the shortest distance if the lines are coplanar?

20. Define shortest distance.

21. Find the equation of the sphere whose centre is (1, 2, 3) and radius is 4.

22. Find the centre of the sphere

$$4x^2 + 4y^2 + 4z^2 - 16x + 4y - 12z + 16 = 0$$

23. Define great circle.

24. Write down the radius of the sphere

$$x^2+y^2+z^2=3$$

25. Define right circular cone.

26. Define guiding curve of a cone.

27. Write down the equation of cone whose vertex is origin and the direction cosines of its generators satisfying the relation

$$4l^2-5m^2+7n^2=0$$

28. Define cylinder.

29. Define normal section in a cylinder.

30. Define radius of a right circular cylinder.

SECTION—B

Answer any *five* of the following questions : 2×5=10

31. Find the equation of the line $y=\sqrt{3}x$ when the axes are rotated through an angle $\pi/3$.

32. If pair of lines $x^2-2pxy-y^2=0$ and $x^2-2qxy-y^2=0$ be such that each pair bisects the angles between the other pair, then prove that $pq+1=0$.

33. Find the radical centre of the three circles

$$x^2+y^2+4x+7=0$$
$$2x^2+2y^2+3x+3y+9=0$$
$$2x^2+2y^2+y=0$$

34. Find the condition that the line $lx+my=n$ is a tangent to the circle $x^2+y^2=a^2$.

35. The normal at the point $(at_1^2, 2at_1)$ meets the parabola again at the point $(at_2^2, 2at_2)$. Prove that $t_2=-t_1-\frac{2}{t_1}$.

36. If e and e' be the eccentricities of a hyperbola and its conjugate, then prove that

$$\frac{1}{e^2}+\frac{1}{e'^2}=1$$

37. Find the equation of plane through the line

$$\frac{x-\alpha}{l}=\frac{y-\beta}{m}=\frac{z-\gamma}{n}$$

and parallel to x-axis.

38. Find the condition that the plane $lx+my+nz=p$ is a tangent plane to the sphere $x^2+y^2+z^2=a^2$.

39. Find the equation of the cone whose vertex is the origin and base is the circle $x = a$, $y^2 + z^2 = b^2$.

40. Find the equation of the cylinder generated by the lines parallel to the z-axis and passing through the curve of intersection of the plane $lx + my + nz = p$ and the surface

$$ax^2 + by^2 + cz^2 = 1$$

SECTION—C

Answer any *five* of the following questions : 5×5=25

41. If by a rotation of the rectangular axes about the origin, the expression $ax^2 + 2hxy + by^2$ changes to $a'x'^2 + 2h'x'y' + b'y'^2$, then prove that $a + b = a' + b'$ and $ab - h^2 = a'b' - h'^2$.

42. Prove that the product of the perpendiculars from the point (x', y') on the line $ax^2 + 2hxy + by^2 = 0$ is

$$\frac{ax'^2 + 2hx'y' + by'^2}{\sqrt{(a-b)^2 + 4h^2}}$$

43. If $S = 0$, $S' = 0$ be two circles of radii r and R, then prove that the circles $\frac{S}{r} \pm \frac{S'}{R} = 0$ will cut orthogonally.

44. Prove that the two circles

$$x^2 + y^2 + 2ax + c^2 = 0 \text{ and } x^2 + y^2 + 2by + c^2 = 0$$

touch, if $\frac{1}{a^2} + \frac{1}{b^2} = \frac{1}{c^2}$.

45. Prove that the line $lx + my = n$ is a normal to the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$, if

$$\frac{a^2}{l^2} + \frac{b^2}{m^2} = \frac{(a^2 - b^2)^2}{n^2}$$

46. Show that the sum of the reciprocals of two perpendicular focal chords of a conic is constant.

47. Prove that the equations of the lines of the shortest distance between the lines

$$\frac{x-2}{1} = \frac{y-7}{-2} = \frac{z-6}{1} \text{ and } \frac{x-6}{7} = \frac{y+7}{-6} = \frac{z}{1}$$

are $11x + 2y - 7z + 6 = 0 = 27x + 26y - 33z + 20$.

48. Find the centre and radius of the circle

$$x^2 + y^2 + z^2 = 49, \quad 2x - y + 3z = 14$$

49. The plane $\frac{x}{a}+\frac{y}{b}+\frac{z}{c}=1$ cuts the axes in A, B, C. Find the equation of the cone whose vertex is the origin and the guiding curve is the circle ABC.

50. Find the equation of the cylinder generated by lines parallel to a fixed line $\frac{x}{l}=\frac{y}{m}=\frac{z}{n}$ and the guiding curve being the conic $z=0$, $ax^2+by^2=1$.

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