TDC (CBCS) Odd Semester Exam., 2022

MATHEMATICS

(5th Semester)

Course No. : MTMSEC–501T

(Integral Calculus)

Full Marks : 50
Pass Marks : 20

Time : 3 hours

The figures in the margin indicate full marks for the questions

UNIT—I

1. Answer any *three* of the following : 1×3=3

(a) Evaluate :

$$\int \frac{\sin^{-1} x}{\sqrt{1-x^2}} dx$$

(b) What do you mean by integration of a function?

(c) Evaluate :

$$\int \frac{2\sin x}{5+3\cos x}\,dx$$

(d) Show that

$$\int \tan x\,dx = \log|\sec x| + c$$

2. Evaluate any *one* of the following : 2

(a) $\int \frac{e^x - 1}{e^x + 1}\,dx$

(b) $\int \frac{dx}{x^2\sqrt{1-x^2}}$

3. Evaluate : 3+2=5

(i) $\int \cos\left(2\cot^{-1}\sqrt{\frac{1-x}{1+x}}\right)dx$

(ii) $\int \frac{x\,dx}{(2x+1)^2}$

Or

Evaluate : 3+2=5

(i) $\int \frac{(\log\sec x)^2}{\cot x}\,dx$

(ii) $\int \frac{\cos^2 x}{\sin^4 x}\,dx$

UNIT—II

4. Answer any *three* of the following : 1×3=3

(a) Express $\int_a^b f(x)\,dx$ as the limit of a sum.

(b) Write down the geometrical interpretation of $\int_a^b f(x)\,dx$.

(c) State the fundamental theorem of integral calculus.

(d) What is the value of $\int_0^{2\pi} f(x)\,dx$, when $f(2a-x) = f(x)$?

5. Answer any *one* of the following : 2

(a) Evaluate $\int_a^b e^{-x}dx$ by the method of summation.

(b) Evaluate :

$$\int_a^b \cos^3\theta\,d\theta$$

6. (a) Evaluate : 3

$$\lim_{n\to\infty}\left\{\frac{1}{n+1} + \frac{1}{n+2} + \cdots + \frac{1}{n+n}\right\}$$

(b) Find the value of

$$\int_0^1 x(\tan^{-1}x)^2\,dx$$ 2

Or

(c) Evaluate : 2

$$\int_{\pi/2}^{\pi/4} \text{cosec}^2 x\,dx$$

(d) Prove that

$$\lim_{n\to\infty} \frac{1^m + 2^m + 3^m + \cdots + n^m}{n^{m+1}} = \frac{1}{m+1}(m-1)$$ 3

UNIT—III

7. Answer any *three* of the following : 1×3=3

(a) Evaluate :

$$\int_0^{\pi/2} \sin^2 x\,dx$$

(b) Show that

$$\int_a^b f(a+b-x)\,dx = \int_a^b f(x)\,dx$$

(c) Let $S_n = \int_0^{\pi/2} \sin^n x\,dx$ and $T_n = \int_0^{\pi/2} \cos^n x\,dx$. Then show that $S_n = T_n$.

(d) Evaluate :

$$\int_0^1 \frac{dx}{x^{2/3}}$$

8. Evaluate any *one* of the following : 2

(a) $\int_0^1 x^6\sqrt{1-x^2}\,dx$

(b) $\int_a^b f(nx)\,dx = \frac{1}{n}\int_{na}^{nb} f(x)\,dx$

9. (a) Prove that

$$\int_0^{\pi} \frac{x\sin x}{1+\cos^2 x}\,dx = \frac{\pi^2}{4}$$ 3

(b) Show that

$$\int_{-\pi/2}^{\pi/2} \sin^7 x\,dx = 0$$ 2

Or

(c) Show that

$$\int_0^{\pi/2} \log\sin x\,dx = \frac{\pi}{2}\log\frac{1}{2}$$ 2

(d) Prove that

$$\int_0^1 \frac{\log(1+x)}{1+x^2}\,dx = \frac{\pi}{8}\log 2$$ 3

UNIT—IV

10. Answer any *three* of the following : 1×3=3

(a) What is the length of the curve $x = f(y)$ from $y = c$ to $y = d$?

(b) Write the parametric equation of an hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$.

(c) Write down the equation of an asteroid.

(d) What is the perimeter of a semicircle?

11. Answer any *one* of the following : 2

(a) Show that complete perimeter of the curve

$$x = \frac{1-t^2}{1+t^2};\ y = \frac{2t}{1+t^2}$$

is 2π.

(b) Write down the equations of cardioid with diagram.

12. Sketch the diagram of the curve

$$\left(\frac{x}{a}\right)^{2/3} + \left(\frac{y}{b}\right)^{2/3} = 1$$

and find its perimeter. 5

Or

Find the length of the arc of the curve

$$x = e^{\theta}\sin\theta;\ y = e^{\theta}\cos\theta$$

from $\theta = 0$ to $\theta = \frac{\pi}{2}$. 5

UNIT—V

13. Answer any *three* of the following : 1×3=3

(a) What is the area of the curve bounded by the curve and the radii vector $\theta = \alpha$ and $\theta = \beta$?

(b) Write down the volume of a cylinder of height h and base radius r.

(c) What is the volume of a sphere generated by the rotation of circle $x^2 + y^2 = 9a^2$?

(d) What is the area of a circle of radius r?

14. Answer any *one* of the following : 2

(a) Find the area of the segment of the parabola $y = (x-1)(4-x)$ cut off by the x-axis.

(b) Find the volume of $y = \sin x$ bounded by the curve and lines $x = 0$, $x = \pi$.

15. Prove that the surface and the volume of the ellipsoid formed by the revolution of $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ round its major axis are $2\pi ab\left\{\sqrt{1-e^2} + \frac{1}{e}\sin^{-1}e\right\}$ and $\frac{4}{3}\pi ab^2$. 5

Or

Prove that the curves $y^2 = 4x$ and $x^2 = 4y$ divide the square bounded by $x = 0$, $x = 4$, $y = 0$, $y = 4$ into three equal areas. 5

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