TDC (CBCS) Odd Semester Exam., 2023

MATHEMATICS

(5th Semester)

Course No. : MTMSEC–501T

(Integral Calculus)

Full Marks : 50
Pass Marks : 20

Time : 3 hours

The figures in the margin indicate full marks for the questions

SECTION—A

Answer *fifteen* questions, selecting any *three* from each Unit : 1×15=15

UNIT—I

1. Evaluate :

$$\int \frac{dx}{ax+b}$$

2. Integrate :

$$\int \sin^2 x \, dx$$

3. Evaluate :

$$\int \log x \, dx$$

4. Show that

$$\int \frac{dx}{x^2 + a^2} = \frac{1}{a} \tan^{-1} \frac{x}{a} + c, \quad a \neq 0$$

UNIT—II

5. State the fundamental theorem of integral calculus.

6. Show that

$$\int_0^{\pi/2} \log \tan x \, dx = 0$$

7. What is the value of

$$\int_{-a}^{a} f(x) dx$$

when $f(-x) = f(x)$?

8. Show that

$$\int_a^b f(a + b - x) dx = \int_a^b f(x) dx$$

UNIT—III

9. Evaluate :

$$\int_0^{\pi/2} \sin^7 x \, dx$$

10. Write down the reduction formula for

$$\int \cos^n x \, dx$$

11. Show that

$$\int_0^{\pi/2} \sin^4 x \cos^5 x \, dx = \frac{8}{315}$$

12. If

$$I_n = \int x^n \cos ax \, dx, \quad J_n = \int x^n \sin ax \, dx$$

then show that

$$a I_n = x^n \sin ax - n J_{n-1}$$

UNIT—IV

13. Find by integration the length of the line segment $y = 2x + 1$ extended from $x = 1$ to $x = 3$.

14. Write the parametric equation of the ellipse

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

15. Write down the equation of cardioid.

16. What is the length of the arc of the circle of radius a and centre at origin, lying in the 1st quadrant?

UNIT—V

17. What is the surface area of a sphere of radius $2r$?

18. Write down the volume of a right circular cone.

19. What is the total volume of the curve $y = f(x)$ revolving about x-axis bounded by $x = a$ to $x = b$?

20. What is the surface area of a cube of length of each side a?

SECTION—B

Answer *five* questions, selecting *one* from each Unit : 2×5=10

UNIT—I

21. Integrate :

$$\int \frac{e^x}{e^x + 1} dx$$

22. Evaluate :

$$\int \frac{dx}{5 + 4\cos x}$$

UNIT—II

23. Show that

$$\int_0^{2a} f(x)\, dx = \int_0^a f(x)\, dx + \int_0^a f(2a - x)\, dx$$

24. Evaluate

$$\int_0^1 x^2\, dx$$

by the method of summation.

UNIT—III

25. If

$$I_n = \int_0^{\pi/4} \tan^n \theta\, d\theta$$

show that

$$I_n + I_{n-2} = \frac{1}{n-1}$$

26. If $m, n \in \mathbb{N}$ and

$$I_{m,n} = \int_0^1 x^{n-1} (\log x)^m \, dx$$

then prove that

$$I_{m,n} = -\frac{m}{n} I_{m-1,n}$$

UNIT—IV

27. Sketch the diagram of the curve $r^2 = a^2 \cos 2\theta$.

28. What do you mean by rectification of plane curve?

UNIT—V

29. Find the volume of the solid generated by revolving about x-axis, the area bounded by the curve $y = 5x - x^2$ and lines $x = 0$, $x = 5$.

30. Write the surface area of the solid formed by revolving the curve $r = a(1 + \cos\theta)$ about the initial line.

SECTION—C

Answer *five* questions, selecting *one* from each Unit : 5×5=25

UNIT—I

31. Integrate the following : 2+3=5

(i) $\int \sec^3 x \, dx$

(ii) $\int \frac{x^2 + 1}{x^4 + 1} dx$

32. Integrate the following : 2+3=5

(i) $\int \sin^{-1} \sqrt{\frac{x}{x+a}} \, dx$

(ii) $\int \sqrt{a^2 - x^2} \, dx$

UNIT—II

33. Show that—

(i) $\int_0^{\pi} \frac{\sin 4x}{\sin x} dx = 0$

(ii) $\int_0^{\pi/4} \log(1 + \tan\theta) \, d\theta = \frac{\pi}{8} \log 2$ 2+3=5

34. Evaluate :

$$\lim_{n \to \infty} \left\{ \frac{1}{n} + \frac{n^2}{(n+1)^3} + \frac{n^2}{(n+2)^3} + \cdots + \frac{1}{8n} \right\}$$

UNIT—III

35. If

$$u_n = \int_0^{\pi/2} x^n \sin x \, dx, \; n > 1$$

then prove that

$$u_n + n(n-1)u_{n-2} = n(\pi/2)^{n-1}$$

36. Prove that

$$\int_0^{\pi/2} \cos^3 x \, dx = \frac{(n-1)(n-3)\cdots 4.2}{n(n-2)\cdots 5.3}$$

if n is odd.

UNIT—IV

37. Find the total length of

$$x^{2/3} + y^{2/3} = a^{2/3}$$

38. Show that the upper half of the curve $r = a(1-\cos\theta)$ is bisected by $\theta = \frac{2\pi}{3}$. Show also that the perimeter of the curve is $8a$.

UNIT—V

39. Find the area of the surface generated by the arc of the parabola $y^2 = 4ax$ bounded by its latus rectum about x-axis.

40. Find the volume of the ellipsoid by the revolution of the ellipse

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

about its major axis.

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