TDC (CBCS) Odd Semester Exam., 2022

MATHEMATICS

(3rd Semester)

Course No. : MTMSEC–301T

Full Marks : 50
Pass Marks : 20

Time : 3 hours

The figures in the margin indicate full marks for the questions

Candidates have to answer from *either* Option—A *or* Option—B *or* Option—C

OPTION—A

Course No. : MTMSEC–301T (A)

(Logic and Sets)

UNIT—I

1. Answer any *three* of the following : 1×3=3

(a) Let p be the statement "Ravi is rich" and q be the statement "Ravi is happy". Write the following statement in symbolic form :

"Ravi is poor or he is both rich and unhappy."

(b) Define proposition.

(c) Check whether the following statement is true or false :
"If the earth is round, then the earth travels around the sun."

(d) Write the negation of the following statement :
"No nice people are dangerous."

2. Answer any *one* of the following : 2

(a) Prove that $p \rightarrow q = \sim p \vee q$.

(b) Show that the following proposition is a tautology :

$$p \rightarrow (p \vee q)$$

3. Answer any *one* of the following : 5

(a) Show that the propositions $p \wedge \sim q$ and $(p \vee q) \wedge (\sim p) \wedge (\sim q)$ are contradiction.

(b) (i) Define tautology and contradiction. 2

(ii) Write the converse, contrapositive and inverse of the following implication : 3
"If it rains today, I will go to college tomorrow."

UNIT—II

4. Answer any *three* of the following : 1×3=3

(a) Form the negation of the following :
"The sum of any two odd integers is an even integer."

(b) Write the following statement using quantifier :
"For all natural numbers, $n+4>3$".

(c) Let $D = \{1, 2, 3, \cdots, 9\}$. Determine the truth value of the following statement :
$\exists x \in D,\ x+4>15$

(d) What is the truth value of the quantification $(\exists x)$, $Q(x)$, if the statement $Q(x)$ and universe of discourse is given as follows?
$Q(x) : x^2 < 12$, U = {positive integer not exceeding 3}

5. Answer any *one* of the following : 2

(a) Write the following predicate in symbolic form :
"Someone in your school has visited Agra."

(b) Let $K(x)$: x is man,

$L(x)$: x is mortal

Express the following using quantifiers :

"All men are mortal."

6. Answer any *one* of the following : 5

(a) (i) Show that $\mathcal{A} : p \rightarrow \sim q$ and $\mathcal{B} : \sim (p \wedge q)$ are logically equivalent. 3

(ii) Translate the following statements into English :

(x) : x is a cat

$A(x)$: x is an animal

1. $\forall x((x) \rightarrow A(x))$
2. $\forall x((x) \wedge \sim A(x))$ 1+1=2

(b) Construct a truth table for

$(p \vee q) \leftrightarrow [(\sim p \wedge r) \rightarrow (q \wedge r)]$

UNIT—III

7. Answer any *three* of the following : 1×3=3

(a) Define empty set. Give an example.

(b) Justify True or False :

$A \cap B = A \cup B \Rightarrow A = B$

(c) Determine the following set :

$\{\phi\} \cap \{a, \phi, \{\phi\}\}$

(d) If $n(A) = 4$ and $n(B) = 8$, then what can be the maximum number of elements in $A \cup B$?

8. Answer any *one* of the following : 2

(a) If A and B are any two sets, find the number of elements in the set $A \cap (A \cup B)^C$.

(b) Prove that

$\{x : |x| \geq 5\} \cap \{y : |y| \leq 5\} = \{-5, 5\}$

9. Answer any *one* of the following : 5

(a) Find the number of integers between 1 and 250 that are divisible by any of the integers 2, 3 and 7.

(b) (i) If $A_i = [0, i]$, where $i \in \mathbb{Z}$, the set of integers, find $A_1 \cup A_2$, $A_3 \cap A_4$ and $\bigcup_{i=5}^{10} A_i$ 3

(ii) If $U = \{1, 2, 3, \cdots, 7\}$, $A = \{1, 2, 3, 5\}$, $B = \{1, 3, 4, 6\}$ and $C = \{1, 2, 4, 7\}$, find $n(A \cap B^C \cap C)$. 2

UNIT—IV

10. Answer any *three* of the following : 1×3=3

(a) Draw the Venn diagram of the symmetric difference of the sets A and B.

(b) Let $A = \{\phi, b\}$, construct the following set $A \cap P(A)$.

(c) For $A = \{a, b, \{a, c\}, \phi\}$, determine the following set $A - \{a, b\}$.

(d) Determine whether the following statement is True or False :

$\{A\} \cup P(A) = P(A)$

11. Answer any *one* of the following : 2

(a) Prove that $A - B = A \cap B^C$.

(b) Prove that $A^C - B^C = B - A$.

12. Answer any *one* of the following : 5

(a) (i) Prove that $A \cap (B \Delta C) = (A \cap B) \Delta (A \cap C)$ 3

(ii) Let $A = \{1, 2, 3, 4, 5\}$ and $B = \{4, 5, 6, 7\}$ find $(A \Delta B) \Delta A$. 2

(b) (i) Let A and B be two arbitrary sets. Show that $P(A \cap B) = P(A) \cap P(B)$ or give counter example. 3

(ii) For all sets A and B, if $A \subseteq B$, then prove that $P(A) \subseteq P(B)$. 2

UNIT—V

13. Answer any *three* of the following : 1×3=3

(a) Give an example of a relation on the set $A = \{a, b, c\}$ which is reflexive, symmetric but not transitive.

(b) Let A be a set with 10 distinct elements. How many relations are there on A?

(c) Define equivalence relation.

(d) Give an example of a partial order relation.

14. Answer any *one* of the following : 2

(a) Given $A = \{1, 3, 5, 7\}$, $B = \{2, 3, 5, 8\}$. List the elements of $(A \cap B) \times (B - A)$.

(b) For integers a and b, define $a \sim b$, if $3a + 4b = 7n$ for some integer n. Find the equivalence class of 0.

15. Answer any *one* of the following : 5

(a) (i) Let $A = \{1, 2, 3\}$. Determine all partitions of A. 2½

(ii) Show that the relation of congruence modulo m on the set $\mathbf{Z}$ of integers is an equivalence relation. 2½

(b) (i) How many elements in $A \times B$ and $B \times A$ are common if n elements are common to A and B? 3

(ii) Give an example of a partially ordered set which has a maximum and a minimum element but is not totally ordered. 2

OPTION—B

Course No. : MTMSEC–301T (B)

(Programming in C)

UNIT—I

1. Answer any *three* of the following : 1×3=3

(a) Write the syntax to define a symbolic constant.

(b) Is float a valid variable name?

(c) What will be the output when the following code is executed?

```
printf("Hello World!");
```

(d) What is a relational expression?

2. Answer any *one* of the following : 2

(a) Write the rules for constructing integer constants.

(b) Distinguish between local and global variables.

3. Answer *either (a)* or *(b)* : 5

(a) Which of the following are invalid variable names and why?

b'day	int	$ hello
@ dot	number	1st

(b) Write a note on C data types.

UNIT—II

4. Answer any *three* of the following : 1×3=3

(a) What are the three logical operators in C ?

(b) What are relational operators?

(c) Determine the value of the logical expression $a > b || a < c$, if $a = 5$, $b = 10$, $c = 7$.

(d) Write the following as a C statement :
$a - b$ is greater than or equal to 10

5. Answer any *one* of the following : 2

(a) What are unary operators? How many operands are associated in a unary operator?

(b) Arrange the following operators based on their hierarchy :
&& || ! ==

6. Answer *either (a)* or *(b)* : 5

(a) Write a C program to find the product of two integers.

(b) Write a note on hierarchy of operators in C.

UNIT—III

7. Answer any *three* of the following : 1×3=3

(a) Write the syntax of if-else statement.

(b) What is the purpose of continue statement?

(c) What is the output when the following loop is executed?

```
while (2 > 1)
    printf("Hello ! \n");
```

(d) If the initial value of 'a' is 5, what will be its value when the following loop is executed?

```
for (i=1, i<=3, ++i)
    a=a*i;
```

8. Answer any *one* of the following : 2

(a) Describe switch statement with a suitable example.

(b) Write a C program to test whether a number is divisible by 3.

9. Answer *either (a)* or *(b)* : 5

(a) Write a note on various conditional statements.

(b) Discuss briefly various loop control statements.

UNIT—IV

10. Answer any *three* of the following : 1×3=3

(a) What is the return type of a function which does not return any value?

(b) What are library functions?

(c) What are recursive functions?

(d) What is the maximum number of arguments a function can have in C ?

11. Answer any *one* of the following : 2

(a) Distinguish between actual and formal arguments with regard to C functions.

(b) Write a short note on function prototypes.

12. Answer *either (a)* or *(b)* : 5

(a) Illustrate two ways of passing arguments to functions in C with suitable examples.

(b) Write a C program to find the factorial of a positive integer using recursion.

UNIT—V

13. Answer any *three* of the following : 1×3=3

(a) Write the syntax to declare an integer array of size 10.

(b) If int $x[3] = \{1, 2, 3\}$, what is the value of $x[1]$?

(c) What should be the type of an array index?

(d) Point out the error, if any, in the following C statement :

```
char name [4]= "Thomas";
```

14. Answer any *one* of the following : 2

(a) In what way does an array differ from an ordinary variable? Explain with an example.

(b) Write a C program to read and display a one-dimensional array.

15. Answer *either (a)* or *(b)* : 5

(a) Write a C program to find the sum of two one-dimensional arrays.

(b) Write a C program to find the largest element in an array.

OPTION—C

Course No. : MTMSEC-301T (C)

(Classical Algebra and Trigonometry)

UNIT—I

1. Answer any *three* of the following : 1×3=3

(a) Define idempotent matrix.

(b) What do you mean by conjugate of a matrix?

(c) Define unitary matrix.

(d) If A be any square matrix, then $A+A^{\theta}$ is Hermitian and $A-A^{\theta}$ is skew-Hermitian.

(Write True of False)

2. Answer any *one* of the following : 2

(a) Prove that a matrix A is involuntary if and only if $(I+A)(I-A)=0$.

(b) If A be a non-singular square matrix, then prove that $(A')^{-1}=(A^{-1})'$.

3. Answer any *one* of the following : 5

(a) If A be an n-rowed square matrix, prove that $\text{adj}\cdot(\text{adj}\cdot A)=|A|^{n-2}\cdot A$, provided $|A|\neq 0$.

(b) Find A^{-1} if the matrix

$$A=\begin{bmatrix}0 & 1 & 2\\ 1 & 2 & 3\\ 3 & 1 & 1\end{bmatrix}$$

UNIT—II

4. Answer any *three* of the following : 1×3=3

(a) What will be the rank of a matrix, if every element of which is unity?

(b) When are a given system of equations said to be consistent?

(c) The rank of a matrix is altered by pre-multiplication or post-multiplication with any non-singular matrix.

(Write True or False)

(d) Define normal form of a matrix.

5. Answer any *one* of the following : 2

(a) Define rank of a matrix.

(b) Find rank of the matrix

$$A=\begin{bmatrix}1 & 3 & 4\\ 2 & 6 & 8\end{bmatrix}$$

6. Answer any *one* of the following : 5

(a) *(i)* Prove that the points (x_1, y_1), (x_2, y_2), (x_3, y_3) are collinear if and only if the rank of the matrix

$$A=\begin{bmatrix}x_1 & y_1 & 1\\ x_2 & y_2 & 1\\ x_3 & y_3 & 1\end{bmatrix}$$

is less than 3. 3

(ii) Examine the consistency of the equations

$$x+y+4z=6$$
$$3x+2y-2z=9$$
$$5x+y+2z=13$$ 2

(b) Solve the following equations :

$$5x-6y+4z=15$$
$$7x+4y-3z=19$$
$$2x+y+6z=46$$

UNIT—III

7. Answer any *three* of the following : 1×3=3

(a) If α, β, γ are the roots of the equation $a_0x^3+a_1x^2+a_2x+a_3=0$, find $\Sigma\alpha\beta$.

(b) Find the sum of the roots of the equation $2x^2-24x+70=0$.

(c) When is a polynomial equation said to be a reciprocal equation?

(d) Every equation of an odd degree has at least one real root of a sign opposite to that of its last term.

(Write True or False)

8. Answer any *one* of the following : 2

(a) Apply Descarte's rule of signs to examine the nature of roots of the equation $x^6+x^4+x^2+x+3=0$.

(b) Find the roots of the equation $x^3=1$.

9. Answer any *one* of the following : 5

(a) *(i)* If α, β, γ are the roots of the equation $x^3+px^2+qx+r=0$, find the equation whose roots are

$$\left(\frac{1}{\beta^2}+\frac{1}{\gamma^2}-\frac{1}{\alpha^2}\right), \left(\frac{1}{\gamma^2}+\frac{1}{\alpha^2}-\frac{1}{\beta^2}\right), \left(\frac{1}{\alpha^2}+\frac{1}{\beta^2}-\frac{1}{\gamma^2}\right).$$ 3

(ii) Solve the equation

$$2x^3 - x^2 - 18x + 9 = 0$$

if two of the roots are equal in magnitude but opposite in sign. 2

(b) (i) If α be an imaginary root of $x^n - 1 = 0$, where n is prime, prove that $(1-\alpha)\cdot(1-\alpha^2)\cdot\cdots\cdot(1-\alpha^{n-1}) = n$. 2

(ii) Prove that the equation $(x+1)^4 = a(x^4+1)$ is a reciprocal equation if $a \neq 1$. 3

UNIT—IV

10. Answer any *three* of the following : 1×3=3

(a) If $x = \cos\theta + i\sin\theta$, find x^2.

(b) Write the expansion of $\cos n\theta$ in terms of $\cos\theta$ and $\sin\theta$, n being a +ve integer.

(c) Find the value of $e^{i\pi}$.

(d) Write the expansion of $(\cos\theta + i\sin\theta)^{-n}$.

11. Answer any *one* of the following : 2

(a) Prove that

$$\left(\frac{1+\cos\phi+i\sin\phi}{1+\cos\phi-i\sin\phi}\right)^n = \cos n\phi + i\sin n\phi$$

(b) Find all possible values of $(-1)^{1/4}$.

12. Answer any *one* of the following : 5

(a) (i) Find the equation whose roots are the nth power of the roots of the equation

$$x^2 - 2x\cos\theta + 1 = 0$$ 3

(ii) If

$$x_r = \left(\cos\frac{\pi}{2^r} + i\sin\frac{\pi}{2^r}\right)$$

then prove that

$$x_1 \cdot x_2 \cdot x_3 \cdot \cdots \cdot \infty = \cos\pi$$ 2

(b) (i) Solve : 3

$$x^6 + x^5 + x^4 + x^3 + x^2 + x + 1 = 0$$

(ii) Find the general value of θ which satisfies the equation

$$(\cos\theta + i\sin\theta)(\cos 2\theta + i\sin 2\theta)\cdots (\cos r\theta + i\sin r\theta) = 1$$ 2

UNIT—V

13. Answer any *three* of the following : 1×3=3

(a) Express $\cos(\alpha - i\beta)$ in the form of $a + ib$.

(b) Write the expansion of $\sinh(x+y)$.

(c) Define hyperbolic function.

(d) Write the Gregory's series if $\tan\theta = x$, $-1 < x < 1$.

14. Answer any *one* of the following : 2

(a) If

$$u = \log \tan\left(\frac{\pi}{4} + \frac{\theta}{2}\right)$$

then prove that $\tanh\frac{u}{2} = \tan\frac{\theta}{2}$.

(b) Prove that

$$i^i = e^{-(4n+1)\pi/2}$$

15. Answer any *one* of the following : 5

(a) If $x < \sqrt{2} - 1$, prove that

$$2\left(x - \frac{x^3}{3} + \frac{x^5}{5} - \cdots \infty\right)$$

$$= \frac{2x}{1-x^2} - \frac{1}{3}\left(\frac{2x}{1-x^2}\right)^3 + \frac{1}{5}\left(\frac{2x}{1-x^2}\right)^5 - \cdots \infty$$

(b) Find the sum

$$\sin\theta - a\sin(\theta+\phi) + \frac{a^2}{2!}\sin(\theta+2\phi) - \frac{a^3}{3!}\sin(\theta+3\phi) + \cdots \infty$$

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