2021/TDC/CBCS/ODD/
MATSEC–301T (A/B/C)/328

TDC (CBCS) Odd Semester Exam., 2021
held in March, 2022

MATHEMATICS

(3rd Semester)

Course No. : MATSEC–301T

Full Marks : 50
Pass Marks : 20

Time : 3 hours

The figures in the margin indicate full marks for the questions

Candidates are to answer *either* Option—A *or* Option—B *or* Option—C

OPTION—A

Course No. : MATSEC–301T (A)

(Logic and Sets)

SECTION—A

Answer any *fifteen* of the following questions : 1×15=15

1. Write the negation of the statement :

p : Every natural number is greater than 0

2. Identify the type of 'Or' used in the following statement and check whether the statement is True or False :

$q : \sqrt{7}$ is a rational number or an irrational number

3. Write down the contrapositive of the statement :

p : If $\frac{a}{b}$ and $\frac{b}{c}$ are integers, then $\frac{a}{c}$ is an integer.

4. Rewrite the following statement so that it is clear that it is an implication :

q : A differentiable function is continuous.

5. Rewrite each of the following with universal and existential quantifiers :

(a) Not all continuous functions are differentiable.

(b) There is no smallest integer.

6. Write the negation of each of the following :

(a) For every real number x, there is an integer n such that $n > x$.

(b) There exists an infinite set whose proper subsets are all finite.

7. If 0 denotes a contradiction, show that

$$p \wedge 0 \Leftrightarrow 0$$

8. If 1 denotes a tautology, show that $p \vee 1 \Leftrightarrow 1$.

9. Justify True or False :

$$A \subseteq B \Rightarrow A^C \subseteq B^C$$

10. What is $A \cap ((A \cap B)^C)$?

11. How many elements are in the power set of the empty set?

12. What is $\mathbb{N} \cap (-5, 5)$?

13. Justify True or False :

$$((A \setminus B) \subseteq (B \setminus A)) \rightarrow (A \subseteq B)$$

14. How many subsets of B of $\{1, 2, 3, \cdots, n\}$ have the property that $B \cap \{1, 2, 3\} = \varnothing$? Explain.

15. If $A = [-4, 4]$ and $B = [0, 5]$, then what is $A \setminus B$ and $B \setminus A$?

16. Prove that $(A \setminus B) \setminus C = A \setminus (B \cup C)$, for any sets A, B and C.

17. Give example of a relation that is neither reflexive nor symmetric nor transitive.

18. Define a partial order relation on a non-empty set.

19. Is every reflexive relation an identity relation? Justify.

20. Define partition of a set.

SECTION—B

Answer any *five* of the following questions : 2×5=10

21. Show that $(p \wedge q) \rightarrow (p \vee q)$ is a tautology.

22. Construct a truth table for the following compound statement :

$$p \rightarrow \sim (q \vee p)$$

23. Show that there is no largest integer.

24. Show that an implication and its contrapositive are logically equivalent.

25. If $A = \varnothing$, find $P(P(P(A)))$.

26. Show that

$$P(A \cap B) = P(A) \cap P(B)$$

27. Prove that

$$\left(\bigcup_{i=1}^{n} A_i \right)^C = \bigcap_{i=1}^{n} A_i^C$$

28. Show that the number of elements in the power set of a set having m elements is 2^m.

29. Determine the partition of $\mathbb{Z}$ produced by the relation 'congruence modulo 5'.

30. Prove that any finite (non-empty) poset must contain maximal and minimal elements.

SECTION—C

Answer any *five* of the following questions : 5×5=25

31. Construct a truth table for the following compound statement : 5

$$(p \vee q) \leftrightarrow [((\sim p) \wedge r) \rightarrow (q \wedge r)]$$

32. (a) Fill in the blanks so that the resulting statement is equivalent to the implication $p \Rightarrow q$: 3

(i) _____ is necessary for _____

(ii) _____ only if _____

(iii) _____ is sufficient for _____

(b) Using the concept of contrapositive, prove that—

"If the average of four different integers is 10, then one of the integers is greater than 11." 2

33. *(a)* Let x be a real number. Show that the following are equivalent :

(i) $x = \pm 1$

(ii) $x^2 = 1$

(iii) If a is any real number, then $ax = \pm a$. 3

(b) Suppose m and n are integers such that $n^2 + 1 = 2m$. Prove that m is the sum of the squares of two integers. 2

34. Using algebra of propositions, establish the following logical equivalences : 3+2=5

(a) $p \to (q \to r) \Leftrightarrow (p \wedge \sim r) \to \sim q$

(b) $p \to (q \vee r) \Leftrightarrow (p \wedge \sim q) \to r$

35. *(a)* If A and B are non-empty sets, show that

$$A \times B = B \times A \text{ iff } A = B$$ 3

(b) Show that

$$A \times B \subseteq C \times D \Rightarrow A \subseteq C \text{ and } B \subseteq D$$ 2

36. *(a)* Show that for any sets A, B and C

$$(A \cap B) \times C = (A \times C) \cap (B \times C)$$ 3

(b) Let $n \geq 1$ be a natural number. How many elements are in the set

$$\{(a, b) \in \mathbb{N} \times \mathbb{N} \,/\, a \leq b \leq n\}?$$

Explain. 2

37. *(a)* Assume that $P(A) = P(B)$, show that $A = B$. 3

(b) Justify if the following is true :

$$P(A) \neq \varnothing \Leftrightarrow A \neq \varnothing$$ 2

38. *(a)* Show that

$$\bigcap_{n=1}^{\infty} \left[0, \frac{1}{n}\right] = \{0\}$$ 3

(b) Justify True or False : 2

$$(A \cup B) \subseteq A \cap B \to A = B$$

39. *(a)* Prove that a poset has atmost one maximum element. 2

(b) Prove that a glb of two elements in a poset $(A, \trianglelefteq)$ is unique whenever it exists. 3

40. *(a)* Show that any two equivalence classes are either disjoint or identical. 2

(b) For natural numbers x and y, define a relation R as $(x, y) \in R$ iff $x^2 + y$ is even. Show that R is an equivalence relation. 3

OPTION—B

Course No. : MATSEC–301T (B)

(Programming in C)

SECTION—A

Answer any *fifteen* of the following questions : 1×15=15

1. Write the syntax for declaring an integer variable x in C.
2. How will you write the arithmetic expression $a^2 + 5a - 7$ in C?
3. Write the general form of scanf statement.
4. Write the syntax of variable declaration.
5. What are relational operators?
6. Write the following as a C expression :

   $x + y$ is less than 5
7. Write the C expression for

   $$x = -b + \sqrt{b^2 - 4ac}$$
8. What are logical operators?

9. Write the general syntax of for loop.
10. What is the purpose of continue statement?
11. Write the general syntax of do-while loop.
12. Give example of an exit-controlled loop.
13. Write the general syntax of function prototype declaration.
14. When is a function defined of void type?
15. What is a recursive function?
16. Can a function have more than one return statement?
17. Write the general syntax for declaring an array.
18. Write the general syntax of initializing a one-dimensional array.
19. If x is an array of size 5, how are the elements of x listed?
20. What is a two-dimensional array?

SECTION—B

Answer any *five* of the following questions : 2×5=10

21. Write the rules for naming variables in C.

22. Write a program to display the words 'Hello World!' on screen.

23. Write a C program to find the area of a rectangle of given sides.

24. Determine the values of each of the following logical expressions, given that $a=5$, $b=10$, $c=-2$:

(a) $a>b||a>c$

(b) $a==c\ \&\&\ b>a$

25. Write a program to display the larger of two given numbers.

26. Explain entry controlled and exit controlled loop.

27. Write a simple program to compute the product of two numbers using a user-defined function.

28. Explain actual arguments and formal arguments with regards to functions in C.

29. Explain the process of initializing a two-dimensional array.

30. Write a note on the uses of arrays in programming.

SECTION—C

Answer any *five* of the following questions : 5×5=25

31. (a) Describe the various types of constants in C. 3

(b) Explain the type definition feature in C. 2

32. Describe the data types in C. 5

33. (a) Write a C program to compute the sum of the squares of three given numbers. 3

(b) Write the rules for precedence of arithmetic operators. 2

34. Explain integer arithmetic, real arithmetic and mixed-mode arithmetic in C. Illustrate with suitable examples. 5

35. Write a C program to compute the sum of the squares of first n natural numbers. 5

36. Explain the use of switch statement with suitable example. 5

37. Write a brief note on user-defined functions, their types, general syntax, and advantages. Illustrate your answer with suitable examples. 5

38. Write a C program to compute the sum of first n natural numbers using function. 5

39. Write a program to find the sum of two one-dimensional arrays entered by the user. 5

40. Write a program to find the largest element in an integer array. 5

OPTION—C

Course No. : MATSEC-301T (C)

(Classical Algebra and Trigonometry)

SECTION—A

Answer any *fifteen* of the following questions : $1\times15=15$

1. If A is a skew-symmetric matrix of odd order, what is the determinant of A? Justify your answer.

2. Define nilpotent matrix.

3. If A is a 4×4 matrix with $|A|=5$, what is the determinant of the adjoint of A?

4. What can you say about the diagonal entries of a skew-Hermitian matrix?

5. What is the rank of the identity matrix of order 5?

6. Define Echelon form of a matrix.

7. If A is a 3×3 non-singular matrix, what is the rank of A^{-1}?

8. What is the condition that a system of linear equations $Ax = B$ is consistent?

9. Find the sum of roots of the equation

$$2x^4 - 5x^3 + x^2 - x + 2022 = 0$$

10. State Descartes' rule of signs.

11. Write the cubic equation, given two of its roots are 1 and $1+i$.

12. Find the equation whose roots are reciprocal of those of $2x^2 + 3x + 1 = 0$.

13. State DeMoivre's theorem.

14. If ω is an imaginary cube root of unity, evaluate $\omega + \omega^2 + \omega^3 + \omega^4 + \omega^5 + \omega^6$.

15. Write the expansion of $\sin\theta$ in ascending powers of θ.

16. Find the value of $e^{i\pi/4}$.

17. Write Gregory's series.

18. Write the formula for the sum of the cosines of n angles in AP.

19. Show that

$$\cosh^2\theta - \sinh^2\theta = 1$$

20. Express $\sin(a+ib)$ in the form $x+iy$ where a, b, x and y are real.

SECTION—B

Answer any *five* of the following questions : 2×5=10

21. Show that the matrix

$$A = \begin{pmatrix} a+ic & -b+id \\ b+id & a-ic \end{pmatrix}$$

is unitary if and only if

$$a^2 + b^2 + c^2 + d^2 = 1$$

22. Find the adjoint of the matrix

$$A = \begin{pmatrix} 1 & 2 & 4 \\ -1 & 0 & 3 \\ 0 & 2 & 1 \end{pmatrix}$$

23. Reduce the matrix to Echelon form

$$A = \begin{pmatrix} 1 & 2 & 1 \\ 6 & 3 & 4 \\ 2 & 1 & 3 \end{pmatrix}$$

24. For what value of α does the system

$$\alpha x + y + 2z = 0$$
$$x + y - z = 0$$
$$2x + 3y = 0$$

has non-trivial solution?

25. Apply Descartes' rule of signs to discuss the nature of roots of the equation

$$x^4 + x^2 + x - 2 = 0$$

26. Solve the equation

$$x^3 - 5x^2 - 16x + 80 = 0$$

given that it has two roots whose sum is zero.

27. Find all possible values of $i^{1/5}$.

28. If $x + \frac{1}{x} = 2\cos\frac{\pi}{7}$, show that

$$x^7 + \frac{1}{x^7} = -2$$

29. Show that

$$\frac{\pi}{8} = \frac{1}{1\times 3} + \frac{1}{5\times 7} + \frac{1}{9\times 11} + \cdots$$

30. If $x + iy = \sin(a + ib)$, show that

$$\frac{x^2}{\cosh^2 b} + \frac{y^2}{\sinh^2 b} = 1$$

SECTION—C

Answer any *five* of the following questions : 5×5=25

31. Prove that every square matrix can be expressed uniquely as the sum of a symmetric and a skew-symmetric matrix. 5

32. *(a)* Show that the inverse of a matrix is unique, if it exists. 3

(b) Check if the matrix

$$A = \frac{1}{3}\begin{pmatrix} -1 & 2 & 2 \\ 2 & -1 & 2 \\ 2 & 2 & -1 \end{pmatrix}$$

is orthogonal. 2

33. Reduce to normal form

$$A=\begin{pmatrix} 1 & 2 & 1 & 0 \\ -2 & 4 & 3 & 0 \\ 1 & 0 & 2 & -7 \end{pmatrix}$$

Hence find its rank. 4+1=5

34. Solve the system of linear equations : 5

$$x+2y+3z=11$$
$$x-2y+4z=3$$
$$x+2y-z=-1$$

35. If the equation $x^3+px^2+qx+r=0$ has roots α, β and γ, find $\sum\alpha^3$ and $\sum\alpha^2\beta$ in terms of p, q and r. 5

36. If α, β and γ are the roots of $x^3+qx+r=0$, then find the equation whose roots are

$$\frac{\beta+\gamma}{\alpha^2},\ \frac{\gamma+\alpha}{\beta^2} \text{ and } \frac{\alpha+\beta}{\gamma^2}$$ 5

37. Prove that

$$\frac{\sin^3\theta}{3!}=\frac{\theta^3}{3!}-\frac{(1+3^2)\theta^5}{\underline{|5}}+(1+3^2+3^4)\frac{\theta^7}{\underline{|7}}+\cdots$$ 5

38. If

$$(1+x)^n=p_0+p_1x+p_2x^2+\cdots$$

show that

(i) $p_0-p_1+p_4-\cdots=2^{n/2}\cos\frac{n\pi}{4}$

(ii) $p_0-p_3+p_4-\cdots=2^{n/2}\sin\frac{n\pi}{4}$

5

39. Find the sum

$$\sqrt{1+\sin\alpha}+\sqrt{1+\sin 2\alpha}+\sqrt{1+\sin 3\alpha}+\cdots+\sqrt{1+\sin n\alpha}$$ 5

40. Prove that

$$\pi=2\sqrt{3}\left[1-\frac{1}{3^2}+\frac{1}{5\times 3^2}-\frac{1}{7\times 3^3}+\cdots\right]$$ 5

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