2023/TDC(CBCS)/ODD/SEM/ MTMSEC-301T/(A/B/C)/309

TDC (CBCS) Odd Semester Exam., 2023

MATHEMATICS

(3rd Semester)

Course No.: MTMSEC-301T

Full Marks: 50
Pass Marks: 20

Time: 3 hours

The figures in the margin indicate full marks for the questions

Candidates have to answer question from either Option—A or Option—B or Option—C

OPTION—A

Course No.: MTMSEC-301T (A)

Logic and Sets)

SECTION—A

Answer fifteen questions, taking any three from each Unit:

remain a Unit—I

1. Write the negation of the following statement:

 $x \in \mathbb{N}$ and 3x + 7 = 0

24J/309

(Turn Over.)

- 2. If p is false, q is true, then what is the truth value of $p \vee -q$?
- 3. If 1 denotes a tautology, then show that $p \lor 1 \Leftrightarrow 1$.
- **4.** Let $A = \{10, 20, 30, 40, 50, 60\}$. How many subsets of A have four elements?

Unit-II

5. Write the following statement using quantifier:

"For all natural numbers, n+3>2."

6. If 0 denotes a contradiction, then show that

$$p \wedge 0 \Rightarrow 0$$

7. Rewrite the following statement as an implication:

"The diagonals of a parallelogram bisect each other."

- 8. Write the negation of each of the following:
 - (a) There exists an infinite set whose power subsets are all finite.
 - (b) For every real number x, there is an integer n such that n > x.

24J/309

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Unit-III

9. Justify True or False:

$$A \subseteq B \Rightarrow A^c \subseteq B^c$$

- 10. What is $N \cap (-3, 7)$?
- 1. If $A = \phi$, then write P(P(A)) in roster form.
- How many subsets of B of $\{1, 2, 3, \dots, n\}$ have the property that $B \cap \{1, 2\} = \emptyset$? Explain.

Unit-IV

- 13. Find the symmetric difference of the sets $A = \{1, 2, 3, 4, 5, 6\}, B = \{2, 4, 6, 8, 10\}.$
- 14. Fill in the blank :

$$A \cap B = B$$
 iff _____.

22 Show as a control of a contrology

- 15. Draw the Venn diagram of $(A \cap B^c \cap C)$ for any sets A, B and C
- **16.** If $A = \phi$, then |P(P(A))| = ?

(Turn Over)

Unit-V

- 17. Define reflexive relation on a set.
- 18. Is every reflexive relation an identity relation?

 Justify.
- 19. Let $X = \{a, b, c, d\}$. Write a relation on X that is symmetric but neither transitive nor reflexive.
- 20. Define maximal element of a poset.

SECTION-B

Answer five questions, taking one from each Unit:

2×5=10

Unit-I

- 21. Construct the truth table for $(-p) \land (q \lor p)$.
- **22.** Show that $(p \land q) \rightarrow (p \lor q)$ is a tautology.

Unit-II

- 23. What is the truth value of the quantification (∃x), Q(x) if the statement Q(x) and universe of discourse is given as follows?
 Q(x):x² <18, U = {positive integer not exceeding 4}</p>
- 24. Show that there is no largest integer.

Unit-III

- **25.** Prove that $(A \setminus B) \setminus C = A \setminus (B \cup C)$ for any sets A, B, C.
- 26. Prove or disprove:

$$A \cap B = \phi \Rightarrow P(A) \cap P(B) = \phi$$

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- **27.** Let $A_n = \{x \in \mathbb{Z} \mid x \ge n\}$ for each $n \in \mathbb{N}$. What is $\bigcap_{n=1}^{\infty} A_n$?
- **28.** Find the value of k for which $A_3 \cap A_5 = A_k$, when $A_n = \{a \in \mathbb{Z} | a \le n\}$.

Unit-V

- 29. Draw the Hasse diagram for the partial order ≤ on {0, 1, 2, 3}.
- 30. Show that the number of elements in the power set of a set having m elements is 2^m .

24J/309

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24J/309

(Turn Over)

SECTION-C

Answer five questions, taking one from each Unit: 5×5=25

Unit-I

- 31. Using algebra of propositions, prove that $(s \Rightarrow ((\neg p \land q) \land r))$ is logically equivalent to $\neg ((p \lor (\neg (q \land r))) \land s)$.
- **32.** (a) Prove that $p \rightarrow q = p \lor q$.
 - (b) Show that an implication and its contrapositive are logically equivalent.

Unit-II

- 33. Using truth table, show that $((p\Rightarrow q)\land (q\Rightarrow r))\Rightarrow (p\Rightarrow r)$ is a tautology.
- 34. Using algebra of propositions, establish the following logical equivalences:
 3+2

(i)
$$p \rightarrow (q \rightarrow r) \Leftrightarrow (p \land \neg r) \rightarrow \neg q$$

(ii)
$$p \rightarrow (q \lor r) \Leftrightarrow (p \land \neg q) \rightarrow r$$

24J/309

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Unit-III

- 35. Show that the union of two finite sets is finite.
- 36. (a) For any sets A, B, C, show that $(A \cap B) \times C = (A \times C) \cap (B \times C)$
 - (b) Show that $P(A \cap B) = P(A) \cap P(B)$.

Unit—IV

37. (a) Show that

$$\bigcap_{n=1}^{\infty} [0, \frac{1}{n}] = \{0\}$$

- (b) If P(A) = P(B), then show that A = B. 2
- 38. (a) If A and B are nonempty sets, then show that $A \times B = B \times A$ iff A = B.
 - (b) Let $n \ge 1$ be a natural number. How many elements are in the set $\{(a, b) \in N \times \mathbb{N} \mid a \le b \le n\}$? Explain.

Unit-V

- 39. State and prove the fundamental theorem of equivalence relations.
- 40. Construct bijection from (1, 2) to (21, 2021). Justify your answer.

24J/309

(Turn Over)

3

3

OPTION-B

Course No.: MTMSEC-301T (B)

(Programming in C)

SECTION-A

Answer fifteen questions, taking any three from each Unit:

Unit-I

- 1. Write the syntax to declare an integer variable.
- 2. How are comments written in a C program?
- 3. Mention two C library functions.
- 4. Write the syntax for scanf statement.

Unit—II

5. Write a C program to print the following line of text:

Welcome to C

6. Convert the following equation to corresponding C statement:

$$A = \frac{(x+y)^2}{6} + xy^2$$

7. Identify the error, if any, in the following C statement:

printf
$$(x = \% d, x)$$
;

8. What are the three logical operators in C?

Unit-III

- 9. Write the syntax of if-else statement.
- 10. What is the purpose of break statement?
- 11. If the initial value of x is 2, then what will be its value after execution of the statement x++;?
- 12. Write the syntax of while loop.

Unit-IV

- 13. Write two advantages of using functions in C.
- 14. What are recursive functions?
- 15. What is the maximum number of arguments a function can have in C?
- 16. What should be the return type of a function that does not return any value?

24J/309

(Turn Over)

24J/309

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Unit-V

- 17. Define an array.
- 18. What should be the type of an array index?
- 19. Point out errors, if any, in the following array declaration:

float y(3);

20. Write the syntax to declare an integer array of size 10.

SECTION-B

Answer five questions, taking one from each Unit

Unit-I

- 21. Write the rules for naming an identifier.
- 22. Distinguish between local and global variables.

Unit-II

23. Write a C program to input two integers and print their sum.

24. What will be the value of x when the following segment is executed?

int x = 10, y = 15;

x = (x < y)?(y + x): y;

Unit-III

- 25. Describe switch statement with a suitable example.
- 26. Write a C program to check whether a number (integer) is divisible by 2.

Unit-IV

- 27. Write a short note on function prototypes.
- 28. Distinguish between actual and formal arguments with regard to C functions.

Unit-V

- 29. What is the difference between an array and an ordinary variable? Explain with an example.
- 30. Write two advantages of using a multidimensional array in a C program.

(Turn Over)

SECTION-C

Answer five questions, taking one from each Unit: 5×5=25

Unit-I

- 31. Write a note on C data types.
- 32. Explain formatted input and output

- 33. Write notes on logical
- logical and relational dimensional to the area and perimeter of a rectangle, taking length and breadth as inputs. 34. Write a C program to compute the area and

- 35. Write a note on various conditional statements in C.
- 36. Discuss briefly about various loop control statements in C.

Unit-IV

- 37. Write a C program to find the factorial of a positive integer using recursion.
- 38. Discuss briefly about various types of functions in C, with suitable examples.

Unit-V

one-dimensional multiand dimensional arrays.

Write a C program to find the least element in an array of integers.

OPTION-C

Course No. : MTMSEC-301T (C)

(Classical Algebra and Trigonometry)

SECTION-A

Answer fifteen questions, taking any three from each Unit: 1×15=15

Unit-I

- 1. Define idempotent matrix.
- 2. Define nilpotent matrix.
- . 3. Define skew symmetric matrix.
- 4. Define unitary matrix.

Unit-II

- 5. Define normal form of a matrix.
- Discuss consistency of a given system of equations.
- 7. What is the rank of zero matrix?
- 8. Define Echelon form of a matrix.

Unit-III

- 9. When is a polynomial equation said to be a reciprocal equation?
- 10. Write down the equation whose roots are 1, -2, 3, -4.
- 11. Write down the sum of the roots of $x^3 px^2 + qx p = 0.$
- 12. Write the cubic equation, given roots are and 1+i.

Unit-IV

- 13. Write Gregory's series.
- 14. Find the value of $e^{i\pi/4}$.
- 15. State de Moiver's theorem.
- 16. If $x = \cos\theta + i\sin\theta$, then find x^2 .

Unit-V

- 17. Expand $\cos(\alpha i\beta)$ in the form of a + ib.
- 18. Define hyperbolic function.
- 19. Define sinh x.
- 20. Write the relation between tan ix and tan hx.

24J/309

(Turn Over)

SECTION-B

Answer five questions, taking one from each Unit:

2×5=10

Unit-I

- 21. If A is a non-singular square matrix, then prove that $(A^1)^{-1} = (A^{-1})^1$.
- 22. Obtain the relation between rank of original matrix and its transpose.

23. Find the rank of the following matrix:

$$A = \begin{bmatrix} 1 & 3 & 4 \\ 2 & 6 & 8 \end{bmatrix}$$

24. Define orthogonality of a square matrix.

Unit-III

- 25. Find the roots of $x^3 = 1$.
- 26. The sum of two roots of

$$x^3 + a_1 x^2 + a_2 x + a_3 = 0$$

is zero. Show that $a_1a_2 - a_3 = 0$.

Unit-IV

- 27. Find all possible values of $(-1)^{1/4}$.
- 28. If

 $\sin \alpha + \sin \beta + \sin \gamma = \cos \alpha + \cos \beta + \cos \gamma = 0$

$$(1+i)^n + (1-i)^n = 2^{(\frac{n}{2}+1)} \cos \frac{n\pi}{4}$$

 $(i) \quad \cos 3\alpha + \cos 3\beta + \cos 3\gamma = 3\cos(\alpha + \beta + \gamma)$ $(ii) \quad \sin 3\alpha + \sin 3\beta + \sin 3\gamma = 3\sin(\alpha + \beta + \gamma)$ Unit - V Unit - V $(1+i)^n + (1-i)^n = 2^{\left(\frac{n}{2}+1\right)}\cos^{\frac{n\pi}{2}}$ $(1+i)^n + (1-i)^n = 2^{\left(\frac{n}{2}+1\right)}\cos^{\frac{n\pi}{2}}$

Answer five questions, taking one from each Unit:

 $5 \times 5 = 25$

31. Find A^{-1} if the matrix

$$A = \begin{bmatrix} 4 & 3 & 8 \\ 6 & 2 & 5 \\ 1 & 5 & 9 \end{bmatrix}$$

32. If

$$A = \begin{pmatrix} \alpha & \beta \\ \gamma & \delta \end{pmatrix}$$

then show that adj(adj A) = A.

Unit-II tant sunse west

33. Solve :

$$x+2y+3z=11$$

$$x+2y+3z=-3$$

$$x+2y+3z=-1$$

34. If A and B are invertible square matrices of the same order, then prove that AB is invertible and $(AB)^{-1} = B^{-1}A^{-1}$.

Unit-III

- 35. If α , β , γ are roots of $x^3 + px^2 + qx + r = 0$, then find the equation whose roots are $\beta + \gamma - 2\alpha$, $\gamma + \alpha - 2\beta$, $\alpha + \beta - 2\gamma$.
- **36.** (a) Solve $2x^3 x^2 18x + 9 = 0$ if two roots are equal in magnitude but opposite on sign.
 - (b) Prove that $(x+1)^4 = \alpha(x^4+1)$ is reciprocal if $\alpha \neq 1$.

Unit-IV

37. (a) If

$$x + \frac{1}{x} = 2\cos\frac{\pi}{7}$$

then show that $x^7 + \frac{1}{x^7} = -2$.

then show that
$$x' + \frac{1}{x^7} = -2$$
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(b) Show that
$$\frac{\pi}{8} = \frac{1}{1 \times 3} + \frac{1}{5 \times 7} + \cdots$$
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Quare matrices we that AB is
$$\frac{x^2}{\cosh^2 b} + \frac{y^2}{\sinh^2 b} = 1$$
Unit—V

39. Find the sum of
$$1 - \frac{1}{3 \cdot 2^2} + \frac{1}{5 \cdot 2^4} - \cdots$$

2

$$\frac{x^2}{\cosh^2 b} + \frac{y^2}{\sinh^2 b} = 1$$

$$1 - \frac{1}{3 \cdot 2^2} + \frac{1}{5 \cdot 2^4} - \cdots$$

40. Prove that

24J-470/309

$$\pi = 2\sqrt{3} \left[1 - \frac{1}{3^2} + \frac{1}{5 \times 3^2} - \frac{1}{7 \times 3^3} + \cdots \right]$$

2023/TDC(CBCS)/ODD/SEM/ MTMSEC-301T/(A/B/C)/309

2

24J/309

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