TDC (CBCS) Even Semester Exam., 2023

MATHEMATICS

(Honours)

(4th Semester)

Course No. : MTMHCC–401T

(Numerical Methods)

Full Marks : 50

Pass Marks : 20

Time : 3 hours

The figures in the margin indicate full marks for the questions

SECTION—A

Answer any *ten* of the following questions : 2×10=20

1. Define relative, absolute, round-off and truncation errors.

2. Compute $\Delta^3(1-2x)(1-3x)(1-4x)$.

3. Prove that $\mu = \left(1+\frac{\delta^2}{4}\right)^{1/2}$.

4. Define interpolation.

5. Construct a forward difference table for $f(x) = x^3 + 2x + 1$, taking $x = 1, 2, 3, 4$.

6. If $f(x) = x^2$, then find the value of $\Delta^3 f(x)$.

7. Write the general quadrature formula for numerical integration.

8. What is the geometrical significance of trapezoidal rule?

9. What is the geometrical significance of Simpson's $\frac{3}{8}$th rule?

10. Write the advantages of Newton-Raphson method.

11. Explain the geometrical significance of regula-falsi method.

12. When can the bisection method be used to find the root of the equation $f(x) = 0$?

13. Explain about pivoting.

14. What is diagonally dominant matrix?

15. Write the sufficient conditions for the convergence of Gauss-Seidel method.

SECTION—B

Answer any *five* of the following questions : 6×5=30

16. *(a)* Round off the numbers 865250 and 37·46235 to four significant digits and compute the absolute and relative errors in each case. 4

(b) Find the absolute error and relative error in $\sqrt{6} + \sqrt{7} + \sqrt{8}$ correct to 4 significant digits. 2

17. *(a)* Prove that $\Delta - \nabla = \Delta\nabla = \delta^2$. 3

(b) Define the shift operator E and show that $E = 1 + \Delta = e^{hD}$, where the interval of difference is h. 3

18. Derive Newton's forward difference interpolation formula. 6

19. *(a)* State Lagrange's interpolation formula. 2

(b) Find the unique polynomial $P(x)$ of degree 2 or less such that $P(1) = 1$, $P(3) = 27$, $P(4) = 64$ by using Lagrange's interpolation formula and also evaluate $P(1 \cdot 5)$. 4

20. State Newton-Cotes quadrature formula and deduce Simpson's $\frac{1}{3}$rd rule for evaluating the integral

$$\int_{x_0}^{x_0 + nh} f(x)dx$$

2+4=6

21. Evaluate

$$\int_0^1 \frac{x^2}{1+x^3}\,dx$$

by using *(i)* trapezoidal rule, *(ii)* Simpson's $\frac{1}{3}$rd rule. Also, compare the errors with the exact value. 3+3=6

22. Show that Newton-Raphson method has second-order convergence. 6

23. *(a)* Find the root of the equation $\cos x = xe^x$, by using the regula-falsi method, correct to four decimal places. 4

(b) Write the sufficient condition for convergence of iteration method. 2

24. Solve the following system of linear equations

$$2x+3y+z=9$$
$$x+2y+3z=6$$
$$3x+y+2z=8$$

by Gauss-Jordan method. 6

25. Consider the system of equations

$$x-ay=b_1$$
$$-ax+y=b_2$$

where a is a real constant. For what values of a, Gauss-Jacobi and Gauss-Seidel methods will converge? 6

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