2021/TDC/CBCS/ODD/
MATHCC–303T/327

TDC (CBCS) Odd Semester Exam., 2021
held in March, 2022

MATHEMATICS

(3rd Semester)

Course No. : MATHCC–303T

(PDE and System of ODE)

Full Marks : 50
Pass Marks : 20

Time : 3 hours

The figures in the margin indicate full marks for the questions

SECTION—A

Answer any *ten* questions : 2×10=20

1. Form the partial differential equation of all spheres whose centre lies on the X-axis.

2. Solve :

$$\frac{\partial^2 z}{\partial x^2} = 0$$

3. Form the partial differential equation by eliminating arbitrary function from the equation

$$z = f(x^2 + y^2)$$

4. Solve $\frac{\partial z}{\partial x} + \frac{\partial z}{\partial y} = z$.

5. Solve $\sqrt{x}\frac{\partial z}{\partial x} + \sqrt{y}\frac{\partial z}{\partial y} = \sqrt{z}$.

6. Solve $a\left(\frac{\partial z}{\partial x} + \frac{\partial z}{\partial y}\right) = z$.

7. Describe the classification of second-order linear partial differential equation.

8. Classify the following PDE :

$$(1+x)^2 \frac{\partial^2 u}{\partial x^2} - 4x\frac{\partial^2 u}{\partial x \partial y} + \frac{\partial^2 u}{\partial y^2} = x$$

9. Solve $\frac{\partial^3 z}{\partial x^3} - 3\frac{\partial^3 z}{\partial x^2 \partial y} + 4\frac{\partial^3 z}{\partial y^3} = 0$.

10. What is boundary value problem?

11. Write down two assumptions for deriving one-dimensional wave equation.

12. Write down one-dimensional wave equation.

13. Show that the ordered pair of functions defined for all t by $(e^{5t}, -3e^{5t})$ is a solution of the system

$$\frac{dx}{dt} = 2x - y \text{ and } \frac{dy}{dt} = 3x + 6y$$

14. If $x = f_1(t)$, $x = f_2(t)$ and $y = g_1(t)$, $y = g_2(t)$ be two solutions of the homogeneous linear system

$$\frac{dx}{dt} = a_{11}(t)x + a_{12}(t)y$$

$$\frac{dy}{dt} = a_{21}(t)x + a_{22}(t)y$$

then write down its solution.

15. Consider the linear system

$$\frac{dx}{dt} = 5x + 3y$$

$$\frac{dy}{dt} = 4x + y$$

Show that $x = 3e^{7t}$, $x = e^{-t}$ and $y = 2e^{7t}$, $y = -2e^{-t}$ are the solutions of this system.

SECTION—B

Answer any *five* questions : 6×5=30

16. *(a)* Form the PDE by eliminating the arbitrary function from the relation

$$z = f(2x+3y) + g(2x+y)$$ 4

(b) Solve : 2

$$\frac{\partial^2 z}{\partial x^2} = \cos x$$

17. *(a)* Find the singular solution of the PDE

$$z = px + qy + p^2 + q^2$$

where $p = \frac{\partial z}{\partial x}, q = \frac{\partial z}{\partial y}$. 4

(b) Find the complete integral of $q + \sin p = 0$, where $p = \frac{\partial z}{\partial x}, q = \frac{\partial z}{\partial y}$. 2

18. Reduce the equation

$$\frac{\partial u}{\partial x} - \frac{\partial u}{\partial y} = u$$

into canonical form and hence find its general solution.

19. Solve by the method of separation variables

$$\frac{\partial u}{\partial x} = 2\frac{\partial u}{\partial t} + u, \; u(x, 0) = 6e^{-3x}$$

20. Reduce the PDE

$$\frac{\partial^2 z}{\partial x^2} - x^2 \frac{\partial^2 z}{\partial y^2} = 0$$

into canonical form and hence solve it.

21. Solve $\frac{\partial^2 z}{\partial x^2} - 2\frac{\partial^2 z}{\partial x \partial y} = x^3 y + e^{2x}$

22. Solve initial boundary value problem

$$u_t = 3u_{xx}, \; u(x, 0) = 17\sin \pi x, \; u(0, t) = u(4, t) = 0$$

23. *(a)* Find all eigenvalues and eigen functions of eigen problem

$$\frac{\partial^2 y}{\partial x^2} + \lambda y = 0, \; y = y(x), \; 0 < x < 1, \; y(0) = y(1) = 0$$ 3

(b) Solve $2u_x + 3u_y = 0, \; u(x, 0) = \sin x$. 3

24. Solve the following system of ordinary differential equations :

$$2\frac{dx}{dt} - 2\frac{dy}{dt} - 3x = t$$

$$2\frac{dx}{dt} + 2\frac{dy}{dt} + 3x + 8y = 2$$

25. Using operator method, find the general solution of the following linear system :

$$\frac{dx}{dt} + \frac{dy}{dt} - x - 2y = 2e^{t}$$

$$\frac{dx}{dt} + \frac{dy}{dt} - 3x - 4y = e^{2t}$$

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