2020/TDC (CBCS)/ODD/SEM/
MTMHCC–303T/329

TDC (CBCS) Odd Semester Exam., 2020
held in March, 2021

MATHEMATICS

(3rd Semester)

Course No. : MTMHCC–303T

(PDE and System of ODE)

Full Marks : 50

Pass Marks : 20

Time : 3 hours

The figures in the margin indicate full marks for the questions

SECTION—A

1. Answer any *ten* of the following questions : 2×10=20

(a) Find a partial differential equation by eliminating a and b from the function

$$z = ax + by + a^2 + b^2$$

(b) Form a partial differential equation by eliminating the arbitrary function f from the relation $z = x + y + f(xy)$.

(c) Solve the equation

$$\frac{\partial^2 z}{\partial x \partial y} = 0$$

(d) Find the general solution of the partial differential equation

$$\frac{\partial^2 z}{\partial y^2} - 2z = 0$$

(e) Solve by the method of separation of variables

$$\frac{\partial z}{\partial x} + \frac{\partial z}{\partial y} = 0$$

(f) Find the canonical form of the partial differential equation

$$2\frac{\partial z}{\partial x} + 3\frac{\partial z}{\partial y} + 8z = 0$$

(g) Solve by using Lagrange's method

$$a\left(\frac{\partial z}{\partial x} + \frac{\partial z}{\partial y}\right) = z$$

(h) What is an integral surface?

(i) Solve :

$$(D^2 - 5DD' + 6D'^2)z = 0$$

(j) Find the particular integral of the partial differential equation

$$(D^2 + D'^2)z = 30(2x + y)$$

(k) Find the characteristics of the partial differential equation

$$\frac{\partial^2 z}{\partial x^2} + 5\frac{\partial^2 z}{\partial x \partial y} + 6\frac{\partial^2 z}{\partial y^2} = 0$$

(l) Describe the classification of second-order linear partial differential equation.

(m) Write down the initial boundary value problem of one-dimensional heat conduction equation.

(n) Write down the initial boundary value problem of one-dimensional wave equation.

(o) Find the eigenvalues of the differential equation

$$\frac{d^2 y}{dx^2} - \lambda^2 y = 0$$

(p) Find the particular integral of the partial differential equation

$$(D^2 + DD' - 2D'^2)z = (2x + y)^{\frac{1}{2}}$$

(q) Define a differential operator.

(r) What is the normal form of a homogeneous system of linear ordinary differential equation?

(s) Solve the differential equation

$$\frac{dx}{dt}+\frac{1}{t}x=e^t$$

(t) What is the matrix form of a system of nonhomogeneous system of linear ordinary differential equation?

SECTION—B

Answer *any* **five** questions

2. Solve the following partial differential equations : 3+3=6

(i) $\frac{\partial z}{\partial y}+2yz=y\sin x$

(ii) $\frac{\partial^2 z}{\partial x \partial y}-\frac{\partial z}{\partial y}=1$

3. Form a partial differential equation by eliminating arbitrary function ϕ from the function $\phi(x^2+y^2+z^2,\ z^2-2xy)=0.$ 6

4. Find the solution of the following partial differential equation using separation of variable method : 3+3=6

(i) $2x\frac{\partial z}{\partial x}+\frac{\partial z}{\partial y}=0$

(ii) $x\frac{\partial z}{\partial x}+3y\frac{\partial z}{\partial y}=0$

5. Find the integral surface of the linear partial differential equation

$$x(y^2+z)\frac{\partial z}{\partial x}-y(x^2+z)\frac{\partial z}{\partial y}=(x^2-y^2)z$$

which contains the straight line $x+y=0$, $z=1$. 6

6. Find the canonical form of the partial differential equation

$$\frac{\partial^2 z}{\partial x^2}=x^2\frac{\partial^2 z}{\partial y^2}$$

and hence solve it. 6

7. Solve the following partial differential equations : 3+3=6

(i) $(D^2+2DD'+D'^2)z=e^{2x+3y}$

(ii) $(4D^2-4DD'+D'^2)z=16\log(x+2y)$

8. Derive the heat conduction equation in a bar. 6

9. Solve the following initial boundary value problem of one-dimensional wave equation by the method of separation of variables : 6

$$\frac{\partial^2 u}{\partial t^2} = c^2 \frac{\partial^2 u}{\partial x^2}, \quad 0 < x < 1, \; 0 < t < \infty$$

$$u(x, 0) = f(x); \; \frac{\partial u}{\partial t}(x, 0) = g(x), \quad 0 < x < 1$$

$$u(0, t) = u(1, t) = 0$$

10. Solve for x and y : 6

$$\frac{dx}{dt} + 2\frac{dy}{dt} - 2x + 2y = 3e^t$$

$$3\frac{dx}{dt} + \frac{dy}{dt} + 2x + y = 4e^{2t}$$

11. Solve the following systems of ordinary differential equations : 3+3=6

(i) $$\frac{dx}{dt} + \frac{dy}{dt} - 2y = 2\cos t - 7\sin t$$

$$\frac{dx}{dt} - \frac{dy}{dt} + 2x = 4\cos t - 3\sin t$$

(ii) $$\frac{dx}{dt} + x = y + e^t; \quad \frac{dy}{dt} + y = x + e^t$$

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