2022/TDC/ODD/SEM/MTMHCC–303T/327

TDC (CBCS) Odd Semester Exam., 2022

MATHEMATICS

(Honours)

(3rd Semester)

Course No. : MTMHCC–303T

(PDE and Systems of ODE)

Full Marks : 50

Pass Marks : 20

Time : 3 hours

The figures in the margin indicate full marks for the questions

UNIT—I

1. Answer any *two* of the following questions : 2×2=4

   (a) Eliminate the arbitrary constants a and b from $z=(x-a)^2+(y-b)^2$ to form the partial differential equation.

   (b) Find a partial differential equation by eliminating a and b from the function $z=ax+by+a^2+b^2$.

   (c) Form the partial differential equation by eliminating the arbitrary function f from $z=f(x^2+y^2)$.

2. Answer any *one* of the following questions : 6

(a) (i) Form the differential equation by eliminating a and b from $z = (x^2 + a)(y^2 + b)$. 3

(ii) Solve $\frac{\partial^2 z}{\partial x \partial y} - \frac{\partial z}{\partial y} = 1$. 3

(b) Form the differential equation by eliminating arbitrary function ϕ from $\phi(x + y + z, x^2 + y^2 - z^2) = 0$.

UNIT—II

3. Answer any *two* of the following questions : 2×2=4

(a) Solve the PDE $\frac{\partial^2 z}{\partial x \partial y} = 0$.

(b) Solve the PDE $\frac{\partial z}{\partial x} + \frac{\partial z}{\partial y} = z$.

(c) What is integral surface?

4. Answer any *one* of the following questions : 6

(a) Solve the PDE

$$\frac{\partial u}{\partial x} = 2\frac{\partial u}{\partial t} + u\,; \quad u(x, 0) = 6e^{-3x}$$

by using the method of separation of variables.

(b) Obtain the integral surface of the linear PDE

$$2y(z-3)\cdot\frac{\partial z}{\partial x} + (2x - z)\cdot\frac{\partial z}{\partial y} = y(2x-3)$$

which passes through the circle $x^2 + y^2 = 2x,\ z = 0$.

UNIT—III

5. Answer any *two* of the following questions : 2×2=4

(a) Describe the classification of second order linear partial differential equation.

(b) Solve $(D^2 - a^2 D'^2)z = 0$.

(c) Find the particular integral of the PDE $(D^2 + D'^2)z = 30(2x + y)$.

6. Answer any *one* of the following questions : 6

(a) Solve the PDE

$$(D^2 + DD' - 6D'^2)z = x^2 \cdot \sin(x + y)$$

(b) Reduce the PDE

$$\frac{\partial^2 z}{\partial x^2} = x^2 \cdot \frac{\partial^2 z}{\partial y^2}$$

into canonical form and hence solve it.

UNIT—IV

7. Answer any *two* of the following questions : 2×2=4

(a) Write the initial value problem of one-dimensional wave equation.

(b) Write the initial value problem of one-dimensional heat equation.

(c) What is boundary value problem?

8. Answer any *one* of the following questions : 6

(a) Derive the one-dimensional homogeneous wave equation.

(b) Solve the IVP

$$\frac{\partial u}{\partial t} = 3 \cdot \frac{\partial^2 u}{\partial x^2} \; ; \; u(x, 0) = 17 \cdot \sin \pi x,$$

$u(0, t) = u(4, t) = 0.$

UNIT—V

9. Answer any *two* of the following questions : 2×2=4

(a) Write the matrix form of a system of non-homogeneous system of linear ordinary differential equations.

(b) Solve :

$$\frac{dx}{dt} = 7x - y$$
$$\frac{dy}{dt} = 2x + 5y$$

(c) Write the normal form of a homogeneous system of linear ordinary differential equations.

10. Answer any *one* of the following questions : 6

(a) Find the general solution of the system

$$\frac{dx}{dt} - \frac{dy}{dt} + 3x = \sin t$$
$$\frac{dx}{dt} + y = \cos t$$

by using operator method.

(b) Solve :

$$\frac{dx}{dt} + 2 \cdot \frac{dy}{dt} - 2x + 2y = 3e^t$$
$$3\frac{dx}{dt} + \frac{dy}{dt} + 2x + y = 4e^{2t}$$

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