2019/TDC/ODD/SEM/
MTMHCC–303T/177

TDC (CBCS) Odd Semester Exam., 2019

MATHEMATICS

(3rd Semester)

Course No. : MTMHCC–303T

(PDE and Systems of ODE)

Full Marks : 50

Pass Marks : 20

Time : 3 hours

The figures in the margin indicate full marks for the questions

UNIT—I

1. Answer any *two* questions from the following : 2×2=4

(a) Find a partial differential equation by eliminating a and b from the equation $z=(x-a)^2+(y-b)^2$.

(b) If $z=f(x-at)+F(x+at)$, show that

$$\frac{\partial^2 z}{\partial t^2}=a^2\frac{\partial^2 z}{\partial x^2}$$

(c) Solve the equation $\frac{\partial z}{\partial x}+2yz=y\sin x$

2. *(a)* Form a partial differential equation by eliminating the arbitrary function ϕ from $\phi(x+y+z, x^2+y^2+z^2)=0$. What is the order of this partial differential equation? 6

Or

(b) Solve the following equations : 3+3=6

(i) $x\dfrac{\partial^2 z}{\partial x\partial y}+\dfrac{\partial z}{\partial y}=4x+2y+z$

(ii) $\dfrac{\partial^2 z}{\partial x\partial y}+\dfrac{\partial z}{\partial x}=2x$

UNIT—II

3. Answer any *two* of the following : 2×2=4

(a) Solve by using Lagrange's method

$$2\frac{\partial z}{\partial x}+3\frac{\partial z}{\partial y}=1$$

(b) Solve by using the method of separable variables :

$$\frac{\partial z}{\partial x}+\frac{\partial z}{\partial y}=0$$

(c) Find the characteristics of the first order linear partial differential equation

$$x^2\frac{\partial z}{\partial x}+y\frac{\partial z}{\partial y}+xyz=1$$

4. *(a)* Solve : 4+2=6

(i) $x(y^2+z)\dfrac{\partial z}{\partial x}-y(x^2+z)\dfrac{\partial z}{\partial y}=z(x^2-y^2)$

(ii) $yz\dfrac{\partial z}{\partial x}+zx\dfrac{\partial z}{\partial y}=xy$

Or

(b) (i) Solve by using the method of separation of variables : 3

$$\frac{\partial^2 z}{\partial x\partial y}+9x^2y^2z^2=0$$

(ii) Find the integral surface of the partial differential equation : 3

$$4yz\frac{\partial z}{\partial x}+\frac{\partial z}{\partial y}+2y=0$$

Passing through the curve $y^2+z^2=1,\ x+z=2$.

UNIT—III

5. Answer any *two* of the following : 2×2=4

(a) Solve $(D^2+2DD'+D'^2)z=0$, where $D\equiv\dfrac{\partial}{\partial x}$ and $D'\equiv\dfrac{\partial}{\partial y}$.

(b) Find the particular integral of the partial differential equation

$$(D^2 + D'^2)z = x + y$$

where $D \equiv \frac{\partial}{\partial x}$ and $D' \equiv \frac{\partial}{\partial y}$.

(c) Find the characteristics of the partial differential equation

$$\frac{\partial^2 z}{\partial x^2} = x^2 \frac{\partial^2 z}{\partial y^2}$$

6. (a) Find the canonical form of the partial differential equation :

$$\frac{\partial^2 z}{\partial x^2} - 9\frac{\partial^2 z}{\partial y^2} = 0$$

and hence solve it. 6

Or

(b) Solve :

$$(D^2 - 6DD' + 9D'^2)z = 12x^2 + 36xy,$$

where $D \equiv \frac{\partial}{\partial x}$ and $D' \equiv \frac{\partial}{\partial z}$ 6

UNIT—IV

7. Answer any *two* of the following : 2×2=4

(a) Solve by using the method of separation of variables :

$$\frac{\partial u}{\partial t} + \frac{\partial u}{\partial x} = 0,\ u(x,0) = 4e^{-x}$$

(b) Give interpretation of the variables x, t and u in the heat conduction equation

$$\frac{\partial u}{\partial t} = k^2 \frac{\partial^2 u}{\partial x^2}$$

(c) Define homogeneous and non-homogeneous boundary condition of a second order partial differential equation.

8. (a) Derive wave equation on a stretched string. 6

Or

(b) Solve the following initial boundary value problem of one-dimensional heat conduction equation by the method of separation of variables : 6

$$\frac{\partial u}{\partial t} = k^2 \frac{\partial^2 u}{\partial x^2},\ 0 < n < L,\ 0 < t < \infty$$

$$u(x, 0) = f(x),\ 0 < x < L$$

$$u(0, t) = u(L, t) = 0,\ 0 < t < \infty$$

UNIT—V

9. Answer any *two* of the following : 2×2=4

(a) Define a differential operator.

(b) What is the normal form of a homogenous system of linear ordinary differential equation?

(c) What is the matrix form of a system of non-homogeneous system of linear ordinary differential equation?

10. Solve (any *one*) : 6

(a) $\frac{dx}{dt} + 2x - 3y = t$

$\frac{dy}{dt} - 3x + 2y = e^{2t}$

(b) $\frac{dx}{dt} - 7x + y = 0$

$\frac{dy}{dt} - 2x - 5y = 0$

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