2023/TDC(CBCS)/ODD/SEM/
MTMHCC–303T/307

TDC (CBCS) Odd Semester Exam., 2023

MATHEMATICS

(Honours)

(3rd Semester)

Course No. : MTMHCC–303T

(PDE and Systems of ODE)

Full Marks : 50

Pass Marks : 20

Time : 3 hours

The figures in the margin indicate full marks for the questions

SECTION—A

Answer *ten* questions, selecting any *two* from each Unit : 2×10=20

UNIT—I

1. Eliminate the arbitrary constants h and k from $(x-h)^2+(y-k)^2+\lambda^2=z^2$ to form a PDE.

2. Form a PDE from $ax^2+by^2+z^2=1$ by eliminating the arbitrary constants a and b.

3. Construct a PDE from the function $u=f(x+ct)+g(x-ct)$ by eliminating f and g.

UNIT—II

4. Obtain the canonical form of the PDE

$$2\frac{\partial z}{\partial x}+3\frac{\partial z}{\partial y}+8z=0$$

5. Solve the PDE

$$2\frac{\partial z}{\partial x}+3\frac{\partial z}{\partial y}=1$$

using Lagrange's method.

6. Solve the PDE

$$\frac{\partial z}{\partial x}+\frac{\partial z}{\partial y}=0$$

by using separation of variables method.

UNIT—III

7. Find the characteristic of the PDE

$$y^2\frac{\partial^2 z}{\partial x^2}-2y\frac{\partial^2 z}{\partial x\partial y}+\frac{\partial^2 z}{\partial y^2}-\frac{\partial z}{\partial x}=6y=0$$

8. Solve the PDE

$$(D^2+2DD'+D'^2)z=0$$

9. Find the particular integral of the PDE

$$(D^2-2DD'+D'^2)z=\sin(2x+3y)$$

UNIT—IV

10. Give interpretation of the variables x, t and u in the heat conduction equation

$$\frac{\partial u}{\partial t}=k^2\frac{\partial^2 u}{\partial x^2}$$

11. Define homogeneous and non-homogeneous boundary conditions of a second-order PDE.

12. Find the eigenvalues of the differential equation

$$\frac{d^2y}{dx^2}-\lambda^2 y=0$$

UNIT—V

13. Define differential operator.

14. Solve :

$$\frac{dx}{dt}+\frac{1}{t}x=e^t$$

15. What is the matrix form of a system of non-homogeneous linear ordinary differential equations?

SECTION—B

Answer *five* questions, selecting *one* from each Unit : 6×5=30

UNIT—I

16. *(a)* Form a PDE by eliminating the arbitrary function ϕ from $z = \phi(\sqrt{x^2 + y^2})$. 3

(b) Construct a PDE from $x + y + z = f(x^2 + y^2 + z^2)$ by eliminating the arbitrary function f. 3

17. *(a)* Solve : 3

$$\frac{\partial z}{\partial y} + 2yz = y \sin x$$

(b) Solve : 3

$$\frac{\partial^2 z}{\partial x \partial y} - \frac{\partial z}{\partial y} = 1$$

UNIT—II

18. *(a)* Solve the following PDE using Lagrange's method : 3

$$\frac{y^2 z}{x} \frac{\partial z}{\partial x} + xz \frac{\partial z}{\partial y} = y^2$$

(b) Solve the following PDE using method of separation of variables : 3

$$\frac{\partial z}{\partial x} - \frac{\partial z}{\partial y} = (x + 2y)z$$

19. Find the integral surface of the PDE

$$x(y^2 + z)\frac{\partial z}{\partial x} - y(x^2 + z)\frac{\partial z}{\partial y} = (x^2 - y^2)z$$

which contains the straight line $x + y = 0$, $z = 1$. 6

UNIT—III

20. *(a)* Solve the PDE

$$(D^2 + 3DD' + 2D'^2)z = x + y$$ 3

(b) Solve the PDE

$$(4D^2 + 4DD' + D'^2)z = 16 \log(x + 2y)$$ 3

21. Find the canonical form of the PDE

$$\frac{\partial^2 z}{\partial x^2} + 5\frac{\partial^2 z}{\partial x \partial y} + 6\frac{\partial^2 z}{\partial y^2} = 0$$

and hence solve it. 6

UNIT—IV

22. Derive heat conduction equation in a bar. 6

23. Solve the PDE

$$\frac{\partial^2 u}{\partial x^2} = \frac{\partial u}{\partial y} + 2u$$

subject to the conditions $u = 0$, $\frac{\partial u}{\partial x} = 1 + e^{-3y}$ when $x = 0$ for all y. 6

UNIT—V

24. Solve the following system of ODEs : 6

$$\frac{dx}{dt} - 7x + y = 0$$

$$\frac{dy}{dt} - 2x - 5y = 0$$

25. Solve the following system of ODEs : 6

$$\frac{dx}{dt} + x = y + e^t$$

$$\frac{dy}{dt} + y = x + e^t$$

★ ★ ★