

2022/TDC/ODD/SEM/ MTMHCC-302T/326

...order -8.

TDC (CBCS) Odd Semester Exam., 2022

MATHEMATICS TOWARD A .S.

Show that what a remark to the Green of the Green

ton ai muser (3rd, Semester) demonstration

Course No.: MTMHCC-302T

Full Marks: 70

Pass Marks: 28

Time: 3 hours

The figures in the margin indicate full marks for the questions

UNIT I THE TOTAL THE TOTAL

1. Answer any two questions of the following:

 $2 \times 2 = 4$

- (a) Write the order of the group of symmetries of a square. Is this group Abelian? 1+1=2
- (b) Write down the elements of the group S_4 , the permutation group on $\{1, 2, 3, 4\}$.

J23/229

(Turn Over)



[Mistagology | Piges MYMMOC-SCRII/826

- (c) Give one example of an Abelian group of order 8 and one non-Abelian group of order 8.
- Answer either (a) and (b) or (c) and (d): $5\times2=10$
 - (a) Let G be any Abelian group and $a, b \in G$. Show that $(ab)^2 = a^2b^2$. Give an example to show that the result is not 2+3=5true if G is non-Abelian.
 - Show that the set $\mathbb{Z}_n := \{0, 1, 2, \dots, n-1\}$ forms a group under addition modulo n. Does $\widetilde{\mathbb{Z}}_n$ form a group under multiplication modulo n? Justify.
 - Prove that a group G is Abelian if and only if $(ab)^{-1} = a^{-1}b^{-1} \ \forall a, b \in G$.
 - (d) Let $G = \{0, 1, 2\}$ and define $a*b = |a-b| \quad \forall a, b \in G$

Construct the composition table for G and examine if G forms a group under *.

The said of the sa

Answer any two questions of the following:

(a) Find the normaliser of $H = \{\pm 1\}$ in $G = \{\pm 1, \pm i\}$. The second $z_i = 1$

(Continued) J23/229

((3))

- (b) Find the centre of S3. Just work (d)
- (c) Prove or disprove : If H and K are subgroups of G, then HK is also a subgroup of G.A. group problem 148
- Answer either (a) and (b) or (c) and (d): (b) but (c) to (d) but (a) will my 5x2=10
 - (a) Show that centre of a group is a subgroup of the group. [11] world [12]
 - (b) Show that intersection of two subgroups of a group is a subgroup of the group. Give an example to show that union of subgroups need not be a subgroup. 3+2=5
 - (c). If H and K are subgroups of G, show that HK is a subgroup of G iff HK = KH. 5
 - (d) Define centraliser of a subgroup in a group. Show that centraliser of any subgroup is again a subgroup. Establish the relation between centraliser and normaliser of a subgroup.

UNIT-III

- 5. Answer any two questions of the following:
 - (a) Give an example of cyclic group of order 6 and a non-cyclic group of order 6.

J23/229

(Turn Over)



((4E))

(b)	S	how	tha	at · a	my	pr	ime	or	der	roup
วาก	is R	bon	ic.	1!	: 91	orq	aib	70	Prove	(c)
									s'S'of'	
	alt	erna	ting	gro	up	$A^{(1)}$	on i	150	23.164	}.

Answer either (a) and (b) or (c) and (d):

(b) a (c) or (c) and (d):

(c) show that centre of a group is a control of a group is a group is a control of a group is a group is a control of a group is a control of a group is a group is a control of a group is a group is a group is a control of a group is a group is

(a) Show that every subgroup of a cyclic aquegroup is cyclic its every subgroup is cyclic its every subgroup of a cyclic

(b) Prove that the disjoint in cycles of in S_n commute with each other in the state of the s

(c) if Show that A_n is a subgroup of S_n and $\frac{1}{N} = \frac{1}{N} \frac{1}{N} = \frac{1}{N} = \frac{1}{N} = \frac{1}{N} = \frac{1}{N} = \frac{1}{N} = \frac{1}{N} \frac{1}{N} = \frac{1}{N} = \frac{1}{N} = \frac{1}{N$

(d) Let G be a finite cyclic group of order n. For d > 0, $d \mid n$, show that G has a unique subgroup of order d.

UNIT-IV

7. Answer any *two* questions of the following: $2 \times 2 = 4$

(a) Can a group of order 12 have a subgroup of order 8? Justify.

J23/229 111

(Continued)

(5)

(b) Find the unit digit in 9223	(b)	Find	the	unit	digit	in	9223
---------------------------------	-----	------	-----	------	-------	----	------

(c) Give an example of a normal subgroup of S_3 and another subgroup of S_3 which is not normal in S_3 .

8. Answer either (a) and (b) or (c) and (d):

5×2=10

5

(a) State and prove Lagrange's theorem.

(b) State Fermat's little theorem. Using it or otherwise, find the remainder when 4²⁹³ is divided by 17.
 2+3=5

(c) Give a proof of Fermat's little theorem using group theory.

(d) Give examples of 5 distinct groups of order 8.

UNIT-V

9. Answer any two questions of the following:

 $2 \times 2 = 4$

(a) Are the groups \mathbb{Z}_4 and $\mathbb{Z}_2 \times \mathbb{Z}_2$ isomorphic? Justify.

- (b) Give an isomorphism between the group of real numbers under addition and the group of positive real numbers under multiplication.
- (c) Show that every normal subgroup of a group must be the kernel of some homomorphism from the group.

J23/229

(Turn Over)

10.	Answer either (a) and (b) or (c) and (d):	×2=10
	(a) State and prove Cayley's theorem.	1+4=5
6 E	(b) Show that the kernel of a group homomorphism is a normal subgroup the group. (c) State and prove first theorem of group.	of 5
8=1 a	(d) Find all group homomorphisms from \mathbb{Z}_4 to \mathbb{Z}_6 .	m 5
	i diet earmane al Salistinal groups of taken and the common of the salistinal groups of the common o	
	A-Ling)	
1-1	never any las questions of the following:	Ų
	Are the groups Z. and Z Z. itemsphie dually.	30
	Oren an isomorphism between the group of real numbers under addition and the group group of real manters under multipasether.	
	Show that every normal subgroup of p	ķa.
	aroz ii aroz sa aroz 2022/TDC/ODD/	SEM/
J23–	-400 /229 MTMHCC-3027	• • •