2023/TDC(CBCS)/EVEN/SEM/ MTMHCC–202T/029

TDC (CBCS) Even Semester Exam., 2023

MATHEMATICS

(Honours)

(2nd Semester)

Course No. : MTMHCC–202T

(Differential Equations)

Full Marks : 50

Pass Marks : 20

Time : 3 hours

The figures in the margin indicate full marks for the questions

SECTION—A

Answer any *ten* of the following questions : 2×10=20

1. Write the degree and order of the following differential equations :

    (i) $k\dfrac{d^2y}{dx^2} = \left[1+\left(\dfrac{dy}{dx}\right)^2\right]^{\frac{3}{2}}$

    (ii) $dy = (x + \sin x)\,dx$

J23/535 (Turn Over)

(2)

2. Obtain a differential equation satisfied by the family of concentric circles.

3. Prove that $\sin 2x$ and $\cos 2x$ are solutions of $y'' + 4y = 0$ and these solutions are linearly independent.

4. Determine the necessary condition for a differential equation of first order and first degree to be exact.

5. Find the integrating factor of

$$(x^2y - 2xy^2)dx - (x^3 - 3x^2y)dy = 0$$

6. Solve :

$$x^2\left(\frac{dy}{dx}\right) + xy = \sqrt{1 - x^2y^2}$$

7. What do you mean by simple compartmental model?

8. Formulate the differential equation for exponential decay model.

9. Write two assumptions for limited growth with harvesting model.

10. Solve :

$$\frac{dx}{xy} = \frac{dy}{y^2} = \frac{dz}{zxy - 2x^2}$$

(Continued)

(3)

11. Show that

$$(2x + y^2 + 2xz)dx + 2xydy + x^2dz = 0$$

is integrable.

12. Solve :

$$zydx = zxdy + y^2dz$$

13. Find the PI of $(D^2 - 3D + 2)y = \cosh x$; $D \equiv \frac{d}{dx}$.

14. Solve :

$$\frac{d^2y}{dx^2} - 3\frac{dy}{dx} + 2y = e^x$$

15. Find PI of $(D^2 + a^2)y = \sin ax$.

SECTION—B

Answer any *five* of the following questions : 6×5=30

16. (a) Find the differential equation of all circles of radius a

$$(x - h)^2 + (y - k)^2 = a^2 \qquad 3$$

   (b) Prove that the functions 1, x, x^2 are linearly independent. Hence form the differential equation whose solutions are 1, x, x^2. 3

(Turn Over)

(4)

17. *(a)* Form the differential equation of the curves $xy = ae^x + be^{-x} + x^2$.   3

    (b) Show that $y = 2\sin x + 3\cos x$ is an explicit solution of the differential equation $\dfrac{d^2 y}{dx^2} + y = 0$, for all real x.   3

18. Define integrating factor. Find the integrating factor of the differential equation

    $(xy \sin xy + \cos xy)\,y\,dx + (xy \sin xy - \cos xy)\,x\,dy = 0$

    and hence solve it.   1+1+4=6

19. *(a)* Solve :   3

    $(1 - x^2)\left(\dfrac{dy}{dx}\right) + 2xy = x\sqrt{1 - x^2}$

    (b) Solve :   3

    $x\left(\dfrac{dy}{dx}\right) + y \log y = xye^x$

20. Discuss the simple compartmental model.   6

21. Let in a lake the pollution level is 7%. If the concentration of the incoming water is 2% and 10000 litres of water per day is allowed to enter the lake, find the time when pollution level is 5% and volume of the lake is 200000 litres. Also, find pollution level after 32 days.   6

J23/535

(5)

22. *(a)* Solve :   3

    $xdx + ydy - \sqrt{(a^2 - x^2 - y^2)}\,dz = 0$

    (b) Solve :   3

    $\dfrac{dx}{dt} - 7x + y = 0$, $\dfrac{dy}{dt} - 2x - 5y = 0$

23. *(a)* Solve :   3

    $yz \log z\,dx - zx \log z\,dy + xy\,dz = 0$

    (b) Solve $\dfrac{dx}{dt} = -\omega y$ and $\dfrac{dy}{dt} = \omega x$. Also, show that the point (x, y) lies on a circle.   3

24. *(a)* Solve :   3

    $\dfrac{d^3 y}{dx^3} - y = (e^x + 1)^2$

    (b) Solve the equation $\dfrac{d^2 y}{dx^2} = a + bx + cx^2$, given that $\dfrac{dy}{dx} = 0$ when $x = 0$ and $y = d$ when $x = 0$.   3

J23/535

25. *(a)* Solve : 2

$$\frac{d^2y}{dx^2} + \frac{1}{x} \cdot \frac{dy}{dx} = \frac{12 \log x}{x^2}$$

(b) Use the method of variation of parameters to solve

$$\frac{d^2y}{dx^2} - 3\frac{dy}{dx} + 2y = \frac{e^x}{1+e^x}$$ 4

★ ★ ★