2019/TDC/EVEN/MTMHC–202T/030

TDC (CBCS) Even Semester Exam., 2019

MATHEMATICS

(2nd Semester)

Course No. : MTMHCC–202T

(Differential Equations)

Full Marks : 50

Pass Marks : 20

Time : 3 hours

The figures in the margin indicate full marks for the questions

1. Answer any *ten* of the following : 1×10=10

(a) Find the order and degree of the differential equation

$$\frac{d^3y}{dx^3}+x^2\left(\frac{d^2y}{dx^2}\right)^3+\frac{dy}{dx}=2$$

(b) What is the order of the differential equation of a three-parameter family of curves?

(c) Obtain the differential equation whose solution is $y=mx+c$, where m is fixed and c is a parameter.

(d) Find the integrating factor of

$$x\frac{dy}{dx}+y=\sin x$$

(e) Is the differential equation

$$(x+y)^2 dx-(y^2-2xy-x^2)dy=0$$

exact?

(f) Solve $xdy = ydx$.

(g) Write the differential equation for diffusion of medicine in bloodstream.

(h) Write the differential equation of simple harmonic motion.

(i) Write the necessary and sufficient condition for integrability of the total differential equation

$$Pdx+Qdy+Rdz=0$$

(j) Solve $xdy-ydx=2x^2zdz$.

(k) Write Bernoulli's differential equation.

(l) Solve $(D-1)^3 y=0$.

(m) Find $\frac{1}{D^2}\cos 2x$.

(n) Find the complementary function of the differential equation

$$\frac{d^2y}{dx^2}+\frac{dy}{dx}+y=xe^x$$

Answer **five** questions, taking **one** from each Unit

UNIT—I

2. (a) Find the differential equation of the family of circles touching the X-axis. 4

(b) Show that $y_1(x)=e^x\sin x$ and $y_2(x)=e^x\cos x$ are solutions of the differential equation

$$\frac{d^2y}{dx^2}-2\frac{dy}{dx}+2y=0$$

Use Wronskian to check, if the solutions are linearly independent or not. 2+2=4

3. (a) Obtain the differential equation whose solution is

$$y=a\cos x+b\sin x+\frac{1}{x}(b\cos x-a\sin x) \quad 4$$

(b) Show that the Wronskian of the functions x^2 and $x^2\log x$ are non-zero. Can these functions be independent solutions of an ordinary differential equation? If so, determine the differential equation. 1+3=4

UNIT—II

4. (a) If the differential equation $Mdx + Ndy = 0$ is homogeneous of degree n and $Mx + Ny \neq 0$, then show that $\frac{1}{Mx + Ny}$ will be integrating factor of the equation. 4

(b) Solve : 4

$$\frac{dy}{dx} = \sin(x+y) + \cos(x+y)$$

5. (a) Solve : 2+2=4

(i) $(1+x^2)\frac{dy}{dx} + y = e^{\tan^{-1} x}$

(ii) $(x+y+1)dx + (x-y)dy = 0$

(b) Solve the differential equation by reducing it to linear form

$$\frac{dz}{dx} + \frac{z}{x}\log z = \frac{z}{x^2}(\log z)^2$$ 4

UNIT—III

6. Discuss the population growth model. Find the time in which (a) the population doubles and (b) the population reduces to half. 6+1+1==8

7. Discuss the simple compartmental model. 8

UNIT—IV

8. (a) Solve : 4

$$\frac{dx}{dt} + 4x + 3y = t$$
$$\frac{dy}{dt} + 2x + 5y = e^t$$

(b) Solve the total differential equation

$$yz(1+x)\,dx + zx(1+y) + xy(1+z)\,dz = 0$$ 4

9. (a) Solve : 5

$$\frac{d^2x}{dt^2} - 3x - 4y + 3 = 0$$
$$\frac{d^2y}{dt^2} + y + x + 5 = 0$$

(b) Test the integrability of the total differential equation

$$(y^2 + yz)\,dx + (z^2 + zx)\,dy + (y^2 - xy)\,dz = 0$$ 3

UNIT—V

10. Solve : 4+4=8

(i) $\frac{d^2y}{dx^2} - y = x\sin x$

(ii) $(x^3D^3 + x^2D^2)y = x$

11. *(a)* Find the particular integral of the differential equation

$$\frac{d^2y}{dx^2}+\frac{dy}{dx}=x^2+2x+4$$ 3

(b) Solve the differential equation

$$\frac{d^2y}{dx^2}-y=\frac{2}{1+e^x}$$

by the method of variation of parameters. 5

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