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2022/TDC(CBCS)/EVEN/SEM/
MTMHCC–201T/256

**TDC (CBCS) Even Semester Exam., 2022**

# MATHEMATICS

( Honours )

**( 2nd Semester )**

Course No. : MTMHCC–201T

**( Real Analysis )**

*Full Marks* : 70

*Pass Marks* : 28

*Time* : 3 hours

*The figures in the margin indicate full marks for the questions*

## SECTION—A

Answer any *ten* questions : 2×10=20

1. If $M$ is a neighbourhood of a point $x$ and $N \supset M$, then show that $N$ is also a neighbourhood of $x$.

2. Show that $\mathbb{N} \times \mathbb{N}$ is countable.

3. Let

$$S = \left\{1 + \frac{(-1)^n}{n},\ n \in \mathbb{N}\right\}$$

Find supremum and infimum of $S$.

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**4.** If $x$ is a limit point of a set $S \subset \mathbb{R}$, is $x$ a limit point of $S-\{x\}$? Justify your answer.

**5.** Define open and closed sets.

**6.** Obtain the derived set of the following sets :

*(i)* $\{1, 2, 3, 4, 5\}$

*(ii)* $\{1, 2, 3, 4, \ldots 500\}$

**7.** Find the limit of the sequence $\{x_n\}$ if

$$x_n = \frac{1^2+2^2+3^2+\cdots+n^2}{n^3}$$

**8.** Give examples of divergent sequences $\{x_n\}$ and $\{y_n\}$ such that $\{x_n+y_n\}$ converges.

**9.** Prove that the limit of a convergent sequence is unique.

**10.** Define subsequence and give example.

**11.** Show that every convergent sequence is a Cauchy sequence.

**12.** State Bolzano-Weierstrass theorem for sequence.





**13.** Prove that if a series converges, its $n$th term must necessarily approach to zero.

**14.** Define alternating series and state when an alternating series is said to be convergent.

**15.** Find $n$th partial sum of the series

$$\frac{1}{1\cdot 3}+\frac{1}{3\cdot 5}+\frac{1}{5\cdot 7}+\cdots\infty$$

## SECTION—B

Answer any *five* questions : 10×5=50

**16.** *(a)* Prove that every open interval in $\mathbb{R}$ is a neighbourhood of each of its points. 2

*(b)* Show that the countable union of countable sets is countable. 4

*(c)* State and prove Archimedean property. 4

**17.** *(a)* Show that every superset of an uncountable set is uncountable. 2

*(b)* Let $A$ and $B$ be two non-empty subsets of $\mathbb{R}$ and let

$$C = \{x+y \mid x \in A \text{ and } y \in B\}$$

Show that $\sup C = \sup A + \sup B$. 4

*(c)* Prove that the set of rational numbers is not order complete. 4

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**18.** *(a)* If $A$ and $B$ are sets of real numbers, then show that

$$(A \cup B)' = A' \cap B'$$ 3

*(b)* Show that union of two open sets is an open set. 3

*(c)* Show that a set is closed if and only if it contains all its limit points. 4

**19.** *(a)* If $A$ and $B$ are sets of real numbers and $A \subset B$, then show that $A' \subset B'$. 2

*(b)* Prove that a set is closed if and only if its complement is open. 4

*(c)* State and prove Bolzano-Weierstrass theorem. 4

**20.** *(a)* Prove that the sequence with $n$th term

$$x_n = \frac{2n-7}{3n+2}$$

is monotonically increasing and bounded. 3

*(b)* If the sequences $\{x_n\}$ and $\{y_n\}$ converge to $x$ and $y$ respectively, then show that the sequence $\{x_n y_n\}$ is convergent and converges to $xy$. 4

*(c)* Show that the sequence $\{x_n\}$, where $x_1 = \sqrt{2}$ and $x_{n+1} = \sqrt{2 + x_n}$ $\forall\, n \geq 1$ is convergent. 3





**21.** *(a)* Prove that every monotonically increasing sequence bounded above is convergent and converges to its supremum. 3

*(b)* Prove that the sequence

$$\left\{\left(1 + \frac{1}{n}\right)^n\right\}$$

is convergent and find its limit. 4

*(c)* Show that the sequence $\{x_n\}$ is convergent where

$$x_n = 2 + (-1)^n \frac{1}{n}$$ 3

**22.** *(a)* Find subsequences of the sequence

$$\left\{\frac{n+1}{n+2}\right\}$$ 3

*(b)* Show that the sequence $\{x_n\}$ defined by

$$x_n = \frac{n}{n+1}$$

is a Cauchy sequence. 3

*(c)* State and prove Cauchy's general principle of convergence for sequence. 4

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23. (a) Prove that the sequence $\{x_n\}$ converges to the limit $l$ if and only if every sub-sequence of $\{x_n\}$ converges to $l$. 4

(b) Prove that every Cauchy sequence is bounded. 3

(c) Using Cauchy's general principle of convergence, show that the sequence $\{x_n\}$ where

$$x_n = 1 + \frac{1}{2} + \frac{1}{3} + \cdots + \frac{1}{n}$$

is not convergent. 3

24. (a) Test the convergence of the following series : 3+3=6

(i) $\frac{1+2}{2^3} + \frac{1+2+3}{3^3} + \frac{1+2+3+4}{4^3} + \cdots \infty$

(ii) $\left(\frac{1}{3}\right)^2 + \left(\frac{1\cdot 2}{3\cdot 5}\right)^2 + \left(\frac{1\cdot 2\cdot 3}{3\cdot 5\cdot 7}\right)^2 + \cdots$

(b) Show that the series

$$\Sigma(-1)^{n-1}\{\sqrt{n^2+1} - n\}$$

is conditionally convergent. 4





25. (a) State and prove ratio test. 4

(b) Test the convergence of the following series :

$$\frac{x}{1\cdot 2} + \frac{x^2}{2\cdot 3} + \frac{x^3}{3\cdot 4} + \cdots$$ 3

(c) Discuss the convergence of the series

$$1 - \frac{1}{2!} + \frac{1}{3!} - \frac{1}{4!} + \cdots$$ 3

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