2023/TDC(CBCS)/EVEN/SEM/
MTMHCC–201T/028

TDC (CBCS) Even Semester Exam., 2023

MATHEMATICS

(Honours)

(2nd Semester)

Course No. : MTMHCC–201T

(Real Analysis)

Full Marks : 70

Pass Marks : 28

Time : 3 hours

The figures in the margin indicate full marks for the questions

SECTION—A

Answer any *ten* questions : 2×10=20

1. Show that $\mathbb{R}$ is not bounded above in $\mathbb{R}$.

2. Show that

$$\bigcap_{n=1}^{\infty} [n, \infty) = \varnothing$$

3. Assuming density of $\mathbb{Q}$ in $\mathbb{R}$, show that $\mathbb{R} \setminus \mathbb{Q}$ is dense in $\mathbb{R}$.

4. Show that 5 is not a limit point of $\mathbb{N}$.

5. Show that $[0, 1)$ is not open in $\mathbb{R}$.

6. Does the set $(-5, 0) \cup \mathbb{N}$ have a limit point in $\mathbb{R}$? Justify.

7. Show that the sequence (x_n), where

$$x_n = (-1)^{n+1} \quad \forall\, n \in \mathbb{N}$$

cannot converge to 1.

8. Find a sequence of irrationals (x_n) such that $x_n \to 0$.

9. Let the sequence (x_n) converges to $x_0 \in \mathbb{R}$. Find an upper bound of the set

$$\{x_n : n \in \mathbb{N}\}$$

10. Prove or disprove :

$$\left(1, \frac{1}{3}, \frac{1}{5}, \frac{1}{9}, \frac{1}{7}, \cdots\right)$$

is a subsequence of the sequence $\left(\frac{1}{n}\right)$.

11. Prove or disprove : Every Cauchy sequence in $\mathbb{R}$ is monotone.

12. Write down four convergent subsequences of the sequence (x_n), where

$$x_n = (-1)^n \quad \forall\, n \in \mathbb{N}$$

13. Let $\sum_{n=1}^{\infty} x_n$ and $\sum_{n=1}^{\infty} y_n$ be convergent series such that $x_n < y_n \;\; \forall\, n \in \mathbb{N}$. Show that

$$\sum_{n=1}^{\infty} x_n < \sum_{n=1}^{\infty} y_n$$

14. Prove or disprove : If the series $\sum_{n=1}^{\infty} x_n$ converges, then $x_n \to 0$.

15. Show that the series $\sum_{n=1}^{\infty} \frac{1}{\underline{|n}}$ converges.

SECTION—B

Answer any *five* questions : 10×5=50

16. *(a)* Show that $\mathbb{Z}$ is a countable set. 5

(b) Let A and B be non-empty bounded below subsets of $\mathbb{R}$. Show that

$$\text{g.l.b.}(A+B) = \text{g.l.b.}(A) + \text{g.l.b.}(B)$$ 5

24. *(a)* Show that $\sum_{n=1}^{\infty} \frac{1}{n^p}$ converges if $p > 1$. 5

(b) If $\sum_{n=1}^{\infty} x_n$ with $x_n > 0 \ \forall n \in \mathbb{N}$, is convergent, then is $\sum_{n=1}^{\infty} x_n^2$ always convergent? Justify your answer. 5

25. *(a)* Let $\sum_{n=1}^{\infty} x_n$ be a convergent series such that $\sum_{n=1}^{\infty} x_n y_n$ is convergent. Is $\sum_{n=1}^{\infty} y_n$ necessarily convergent? Justify. Is $\sum_{n=1}^{\infty} y_n$ necessarily divergent? Justify. 4

(b) Let (x_n) be a sequence in $\mathbb{R}$ such that

$$\sum_{n=1}^{\infty} |x_n|$$

converges. Show that $\sum_{n=1}^{\infty} x_n$ is convergent. Does convergence of $\sum_{n=1}^{\infty} x_n$ necessarily imply the convergence of $\sum_{n=1}^{\infty} |x_n|$? Justify. 3+3=6

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