TDC (CBCS) Odd Semester Exam., 2018

MATHEMATICS

(1st Semester)

Course No. : MTMHCC–102T

(Higher Algebra)

Full Marks : 70
Pass Marks : 28

Time : 3 hours

The figures in the margin indicate full marks for the questions

UNIT—I

1. Answer any *two* from the following questions : $2 \times 2 = 4$

(a) State De Moivre's theorem for rational indices.

(b) Write down the polar form of the complex number $-1+i$.

(c) If $2\cos\theta = a + \frac{1}{a}$, $2\cos\phi = b + \frac{1}{b}$, then find the value of $ab + \frac{1}{ab}$.

2. Answer any *two* from the following questions : 5×2=10

(a) If $(a_1 + ib_1)(a_2 + ib_2)\cdots(a_n + ib_n) = A + iB$, prove that—

(i) $\tan^{-1}\frac{b_1}{a_1} + \tan^{-1}\frac{b_2}{a_2} + \cdots + \tan^{-1}\frac{b_n}{a_n} = \tan^{-1}\frac{B}{A}$;

(ii) $(a_1^2 + b_1^2)\times(a_2^2 + b_2^2)\cdots(a_n^2 + b_n^2) = A^2 + B^2$.

(b) If $(1+x)^n = a_0 + a_1x + a_2x^2 + a_3x^3 + \cdots$ then show that—

(i) $a_0 - a_2 + a_4 - \cdots = 2^{n/2}\cos\frac{n\pi}{4}$;

(ii) $a_1 - a_3 + a_5 - \cdots = 2^{n/2}\sin\frac{n\pi}{4}$.

(c) Expand $\sin\alpha$ in ascending power of α.

(d) If $-\frac{\pi}{4} \le \theta \le \frac{\pi}{4}$, then expand θ in powers of $\tan\theta$.

UNIT—II

3. Answer any *two* from the following questions : 2×2=4

(a) Define equivalence relation on a set.

(b) Let A and B be two sets. If $f : A \to B$ is one-one onto, then prove that $f^{-1} : B \to A$ is also one-one onto.

(c) Give an example of a relation which is reflexive and transitive but not symmetric.

4. Answer any *two* from the following questions : 5×2=10

(a) On the set Z of all integers, consider the relation $R = \{(a, b) : a - b \text{ is divisible by } 3\}$. Show that R is an equivalence relation. Find the partitioning of Z into mutually disjoint equivalence classes.

(b) Let $A = R - \{\frac{3}{5}\}$, $B = R - \{\frac{7}{5}\}$. Also let

$f : A \to B : f(x) = \frac{7x+4}{5x-3}$ and $g : B \to A : g(y) = \frac{3y+4}{5y-7}$

Show that

$(g \circ f) = I_A$ and $(f \circ g) = I_B$ 5

(c) Let $f : R \to R : f(x) = 4x + 3$, for all $x \in R$. Show that f is invertible and find f^{-1}.

(d) If $f : A \to B$ is one-one onto, then prove that f is an invertible function.

UNIT—III

5. Answer any *two* from the following questions : 2×2=4

(a) State the principle of mathematical induction.

(b) Define Euclidean algorithm.

(c) State the fundamental theorem of arithmetic.

6. Answer any *two* from the following questions : 5×2=10

(a) Prove by principle of mathematical induction

$$1 \cdot 2 + 2 \cdot 2^2 + 3 \cdot 2^3 + \cdots + n \cdot 2^n = (n-1)2^{n+1} + 2$$

(b) Prove by principle of mathematical induction

$$1(1!) + 2(2!) + 3(3!) + \cdots + n(n!) = (n+1)! - 1$$

(c) Using division algorithms, prove that the square of any odd integer is of the form $8k+1$ for some integer k.

(d) If $(a, 4) = 2$ and $(b, 4) = 2$, then prove that $(a+b, 4) = 4$.

UNIT—IV

7. Answer any *two* from the following questions : 2×2=4

(a) Examine the nature of the roots of the following equation by using Descartes' rule of signs $x^4 + 2x^2 + 3x - 1 = 0$.

(b) If α, β, γ be the roots of the equation

$$a_0 x^3 + 3a_1 x^2 + 3a_2 x + a_3 = 0$$

then find the value of $\sum \alpha^2 \beta$.

(c) Remove the fractional coefficient of the equation

$$x^4 - \frac{2}{3}x^3 - \frac{5}{6}x^2 + \frac{7}{72}x + \frac{11}{216} = 0$$

8. Answer any *two* from the following questions : 5×2=10

(a) If α, β, γ be the roots of the equation $x^3 + px + r = 0$, then find the value of $\sum \frac{1}{\alpha^2 - \beta\gamma}$.

(b) Solve the following equation by Cardan's method :
$$x^3 - 6x - 4 = 0$$

(c) Solve the equation
$$27x^3 + 42x^2 - 28x - 8 = 0$$
whose roots are in geometric progression.

(d) If α, β, γ be the roots of $2x^3 + x^2 + x + 1 = 0$, then find the equation whose roots are
$$\frac{1}{\beta^3} + \frac{1}{\gamma^3} - \frac{1}{\alpha^3}, \frac{1}{\gamma^3} + \frac{1}{\alpha^3} - \frac{1}{\beta^3}, \frac{1}{\alpha^3} + \frac{1}{\beta^3} - \frac{1}{\gamma^3}$$

UNIT—V

9. Answer any *two* from the following questions : 2×2=4

(a) Define rank of a matrix.

(b) Under what condition the rank of the matrix
$$\begin{bmatrix} 2 & 4 & 2 \\ 2 & 1 & 2 \\ 1 & 0 & x \end{bmatrix}$$
is 3?

(c) If A is a non-zero column matrix and B is a non-zero row matrix, then show that rank $(AB) = 1$.

(Continued)

10. Answer any *two* from the following questions : 5×2=10

(a) Find the rank of the matrix
$$A = \begin{pmatrix} 1 & -1 & 3 & 6 \\ 1 & 3 & -3 & -4 \\ 5 & 3 & 3 & 11 \end{pmatrix}$$
by reducing it to echelon form.

(b) Solve the following system of equations by Gaussian elimination method :
$$5x - 2y + z = 4$$
$$7x + y - 5z = 8$$
$$3x + 7y + 4z = 10$$

(c) Prove that the interchange of a pair of rows does not alter the rank of a matrix.

(d) Reduce the following matrix to normal form :
$$A = \begin{pmatrix} 1 & 2 & 1 & 0 \\ -2 & 4 & 3 & 0 \\ 1 & 0 & 2 & -8 \end{pmatrix}$$

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