2022/TDC/ODD/SEM/
MTMHCC-101T/324

TDC (CBCS) Odd Semester Exam., 2022

MATHEMATICS

(Honours)

(1st Semester)

Course No. : MTMHCC-101T

(Calculus)

Full Marks : 50

Pass Marks : 20

Time : 3 hours

The figures in the margin indicate full marks for the questions

UNIT—I

1. Answer any *two* of the following : 2×2=4

(a) Find y_n, if $y = \sin^3 x$.

(b) If $I_n = \dfrac{d^n}{dx^n}(x^n \log x)$, prove that

$$I_n = nI_{n-1} + (n-1)!$$

(c) Find $\dfrac{dy}{dx}$, where $y = \operatorname{sech} x$.

2. Answer *either* [(*a*) and (*b*)] *or* [(*c*) and (*d*)] :

(*a*) If $y = x^{n-1} \log x$, show that

$$y_n = \frac{\lfloor n-1}{x}$$ 3

(*b*) If $y = e^{-x} \cos x$, prove that $y_4 + 4y = 0$. 3

(*c*) If $u = \sin ax + \cos ax$, show that

$$u_n = a^n \{1 + (-1)^n \sin 2ax\}^{1/2}$$ 3

(*d*) If $y = \frac{\log x}{x}$, prove that

$$y_n = \frac{(-1)^n \lfloor n}{x^{n+1}} \left\{ \log x - 1 - \frac{1}{2} - \frac{1}{3} - \dots - \frac{1}{n} \right\}$$ 3

UNIT—II

3. Answer any *two* of the following : 2×2=4

(*a*) Evaluate :

$$\lim_{x \to 1} \left(\frac{x}{x-1} - \frac{1}{\log x} \right)$$

(*b*) Find the points of inflexion on the curve $y = (\log x)^3$.

(*c*) Find the asymptotes parallel to coordinate axes of the curve

$$(x^2 + y^2)x - ay^2 = 0$$

4. Answer *either* [(*a*) and (*b*)] *or* [(*c*) and (*d*)] :

(*a*) Evaluate : 1½+1½=3

(*i*) $\lim_{x \to \frac{\pi}{2}} \frac{\tan 3x}{\tan x}$

(*ii*) $\lim_{x \to 0} (\cos x)^{\frac{1}{x^2}}$

(*b*) Trace the curve, $y = x^3 - 12x - 16$. 3

(*c*) Show that the curve $(a^2 + x^2)y = a^2 x$ has three points of inflexion. 3

(*d*) Trace the curve $r = a(1 + \cos\theta)$. 3

UNIT—III

5. Answer any *two* of the following : 2×2=4

(*a*) Obtain a reduction formula for

$$\int \sec^n x \, dx$$

(*b*) Evaluate :

$$\int_0^{\pi/2} \sin^8 x \cos^6 x \, dx$$

(*c*) Show that

$$\int x^3 e^{ax} dx = \frac{e^{ax}}{a^4}(a^3x^3 - 3a^2x^2 + 6ax - 6)$$

6. Answer *either* [(a) and (b)] *or* [(c) and (d)] :

(a) If $I_{m,n} = \int \cos^m x \sin^n x\,dx$, then show that

$$(m+n)(m+n-2)\, I_{m,n} = \{(n-1)\sin^2 x - (m-1)\cos^2 x\}$$
$$\cos^{m-1} x \sin^{n-1} x + (m-1)(n-1)\, I_{m-2,\,n-2}$$ 3

(b) If $I_n = \int x^n \cos bx\,dx$ and $J_n = \int x^n \sin bx\,dx$, then show that

$$bI_n = x^n \sin bx - n J_{n-1}$$ 3

(c) If $U_n = \int_0^{\pi/2} x^n \sin x\,dx$, $n > 0$, then show that

$$U_n + n(n-1)U_{n-2} = n\left(\frac{\pi}{2}\right)^{n-1}$$ 3

(d) Show that, if $I_n = \int_0^{\pi/2} \cos^n x\,dx$, then

$$I_n = \frac{n-1}{n} I_{n-2} \quad (n > 2)$$ 3

UNIT—IV

7. Answer any *two* of the following : 2×2=4

(a) Find the length of arc of the parabola $y^2 = 16x$ measured from the vertex to an extremity of the latus rectum.

(b) Find the volume generated by the revolution about x-axis of the area bounded by the loop of the curve $y^2 = x^2(2-x)$.

(c) What do you mean by rectification of plane curve? Write the formula to find the length of the curve $y = f(x)$ from $x = a$ to $x = b$.

8. Answer *either* [(a) and (b)] *or* [(c) and (d)] :

(a) The circle $x^2 + y^2 = a^2$ revolves around the axis. Find the surface area and the volume of the whole surface generated. 3

(b) Show that the length of the arc of the parabola $y^2 = 4ax$, which is intercepted between the points of intersection of the parabola and the straight line $3y = 8x$ is

$$a\left(\log 2 + \frac{15}{16}\right)$$ 3

(c) Show that the area common to the two ellipses $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ and $\frac{x^2}{b^2} + \frac{y^2}{a^2} = 1$ $(a > b)$ is $2ab \tan^{-1} \frac{2ab}{a^2 - b^2}$. 3

(d) If the curve $r = a + b\cos\theta$ $(a > b)$ revolves about the initial line, then show that the volume generated is

$$\frac{4}{3}\pi a(a^2 + b^2)$$ 3

UNIT—V

9. Answer any *two* of the following : 2×2=4

(a) Prove that

$$[\vec{a}\times\vec{b}\ \ \vec{b}\times\vec{c}\ \ \vec{c}\times\vec{a}] = [\vec{a}\,\vec{b}\,\vec{c}]^2$$

(b) If $\frac{d\vec{a}}{dt} = \vec{r}\times\vec{a}$ and $\frac{d\vec{b}}{dt} = \vec{r}\times\vec{b}$, then show that

$$\frac{d}{dt}(\vec{a}\times\vec{b}) = \vec{r}\times(\vec{a}\times\vec{b})$$

(c) Find the vector equation of the plane passing through the origin and parallel to the vectors $\vec{a} = (2, 3, 1)$ and $\vec{b} = (3, -1, 4)$.

10. Answer *either* [(a) and (b)] *or* [(c) and (d)] :

(a) Obtain the equation of a sphere having the points $\vec{a}$ and $\vec{b}$ as the extremities of a diameter. 3

(b) Find the equation of the plane through the point $2\hat{i} + 3\hat{j} - \hat{k}$ and perpendicular to the vector $3\hat{i} - 4\hat{j} + 7\hat{k}$. Also find the perpendicular distance from origin to this plane. 2+1=3

(c) Prove that the derivative of a vector of constant magnitude is perpendicular to the vector. 2

(d) Find the vector equation to the line joining the points $(7, -3, 4)$ and $(2, -1, 1)$, and determine the point where it cuts the plane through $(2, 1, -3)$, $(4, -1, 2)$ and $(3, 0, 1)$. 4

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