TDC (CBCS) Odd Semester Exam., 2019

MATHEMATICS

(1st Semester)

Course No. : MTMHCC–101T

(Calculus)

Full Marks : 50

Pass Marks : 20

Time : 3 hours

The figures in the margin indicate full marks for the questions

UNIT—I

1. Answer any *two* of the following : 2×2=4

(a) Find y_n, if $y = \sin 3x \cos 2x$.

(b) If $y = a\cos(\log x) + b\sin(\log x)$, show that $x^2 y_2 + xy_1 + y = 0$.

(c) Using Leibnitz's theorem, differentiate n times the equation

$$(1 + x^2) y_2 + (2x - 1) y_1 = 0$$

2. Answer *either* (a) and (b) *or* (c) and (d) :

(a) If $y = x^{2n}$, where n is a +ve integer, show that

$$y_n = 2^n \{1 \cdot 3 \cdot 5 \cdots (2n-1)\} x^n$$ 3

(b) If $y = \tan^{-1} x$, then prove that

$$(1+x^2) y_{n+1} + 2nxy_n + n(n-1)_{n-1} = 0$$

Find also the value of $(y_n)_0$. 3

(c) If $y = e^{-x} \cos x$, prove that

$$y_4 + 4y = 0$$ 3

(d) By forming in two different ways the nth derivative of x^{2n}, show that

$$1 + \frac{n^2}{1^2} + \frac{n^2(n-1)^2}{1^2 \cdot 2^2} + \frac{n^2(n-1)^2(n-2)^2}{1^2 \cdot 2^2 \cdot 3^2} + \cdots = \frac{(2n)!}{(n!)^2}$$ 3

UNIT—II

3. Answer any *two* of the following : 2×2=4

(a) Evaluate :

$$\underset{n\to\infty}{\text{Lt}} \frac{x^4}{e^x}$$

(b) Define inflection point on a curve. Give example to show that a point where $y^n = 0$ is not necessarily an inflection point.

(c) Write the parametric and Cartesian equation of an astroid and draw a rough sketch.

4. Answer *either* (a) and (b) *or* (c) and (d) :

(a) Find—

(i) $\underset{x\to 0}{\text{Lt}} \left(\frac{\sin x}{x}\right)^{\frac{1}{x}}$;

(ii) $\underset{x\to \pi/2}{\text{Lt}} (1-\sin x)\tan x$. 1½+1½=3

(b) Examine the curve $y = -2x^3 + 6x^2 - 3$ for concavity and points of inflection, if any. Also draw a rough sketch. 3

(c) Evaluate : 1½+1½=3

(i) $\underset{x\to 0}{\text{Lt}} \left(\frac{\tan x}{x}\right)^{\frac{1}{x}}$

(ii) $\underset{x\to 1}{\text{Lt}} \left(\frac{1}{x^2-1} - \frac{2}{x^4-1}\right)$

(d) Define horizontal asymptote of a curve. Find the asymptotes of the curve

$$y = \frac{x+3}{x+2}$$ 3

UNIT—III

5. Answer any *two* of the following : 2×2=4

(a) Obtain a reduction formula for

$$\int (\log x)^n \, dx$$

(b) Evaluate :

$$\int_0^{\pi/2} \cos^8 \theta \, d\theta$$

(c) If

$$I_n = \int_0^{\pi/2} \sin^n x \, dx$$

n being a +ve integer, show that

$$I_n = \frac{n-1}{n} I_{n-2}$$

6. Answer *either* (a) and (b) *or* (c) and (d) :

(a) If

$$I_{m,n} = \int \sin^m x \cos nx \, dx$$

show that

$$(n^2 - m^2) I_{m,n} = \sin^{m-1} x (n \sin nx \sin x + m \cos nx \cos x) - m(m-1) I_{m-2,n}$$ 3

(b) If

$$U_n = \int_0^{\pi/2} x^n \sin x \, dx, \quad n > 0$$

show that

$$U_n + n(n-1) U_{n-2} = n\left(\frac{\pi}{2}\right)^{n-1}$$ 3

(c) Obtain a reduction formula for

$$I_{m,n} = \int \cos^m x \cos nx \, dx$$

connecting with $I_{m-1,\, n-1}$. 3

(d) Prove that

$$\int_0^{\pi/2} \sin^n x \, dx$$

$$= \frac{(n-1)(n-3)\cdots 4\cdot 2}{n(n-2)\cdots 5\cdot 3}, \text{ if } n \text{ is odd}$$

$$= \frac{(n-1)(n-3)\cdots 3\cdot 1}{n(n-2)\cdots 4\cdot 2}\cdot\frac{\pi}{2}, \text{ if } n \text{ is even}$$ 3

UNIT—IV

7. Answer any *two* of the following : 2×2=4

(a) What do you mean by rectification of plane curve? Write the formula to find the length of the curve $y = f(x)$ from $x = a$ to $x = b$.

(b) Determine the area bounded by the parabola $y^2 = 4ax$ and its latus rectum.

(c) Find the surface area of a solid generated by revolving the semicircular arc of radius c about the axis of x.

8. Answer *either* (a) and (b) *or* (c) and (d) :

(a) Show that the perimeter of the curve $r = a(1 - \cos\theta)$ is $8a$. 3

(b) Show that the area cut-off a parabola by any double ordinate is two-third of the corresponding rectangle contained by the double ordinate and its distance from the vertex. 3

(c) Find the area of the astroid

$$x^{2/3} + y^{2/3} = a^{2/3}$$
3

(d) Find the surface area of the solid generated by revolving the cycloid $x = a(\theta + \sin\theta)$, $y = a(1 + \cos\theta)$ about its base. 3

UNIT—V

9. Answer any *two* of the following : 2×2=4

(a) Show that $\vec{a} \times (\vec{b} \times \vec{c})$, $\vec{b} \times (\vec{c} \times \vec{a})$ and $\vec{c} \times (\vec{a} \times \vec{b})$ are coplanar.

(b) Find the vector equation of the line parallel to the vector $\hat{i}$ and passing through the point (0, 1, 0).

(c) Show that the derivative of a constant vector is zero.

10. Answer *either* (a) and (b) *or* (c) and (d) :

(a) Prove that $[\vec{a} \times \vec{b} \;\; \vec{b} \times \vec{c} \;\; \vec{c} \times \vec{a}]$

$$= \begin{vmatrix} \vec{a} \cdot \vec{a} & \vec{a} \cdot \vec{b} & \vec{a} \cdot \vec{c} \\ \vec{b} \cdot \vec{a} & \vec{b} \cdot \vec{b} & \vec{b} \cdot \vec{b} \\ \vec{c} \cdot \vec{a} & \vec{c} \cdot \vec{b} & \vec{c} \cdot \vec{c} \end{vmatrix}$$

$$= [\vec{a} \; \vec{b} \; \vec{c}]^2$$
3

(b) Find the vector equation of a plane passing through three given points. 3

(c) If $\vec{r} = 3t\hat{i} + 3t^2\hat{j} + 2t^3\hat{k}$, where t is a scalar, show that

$$\left[\frac{d\vec{r}}{dt}\ \frac{d^2\vec{r}}{dt^2}\ \frac{d^3\vec{r}}{dt^3}\right] = 216$$ 3

(d) If $\hat{i} + \hat{j} + 2\hat{k}$ and $2\hat{i} + \hat{j} - \hat{k}$ are position vectors of the extremities of a diameter of a sphere, find the equation of the sphere both in vector and Cartesian forms. 3

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