2018/TDC/ODD/MTMC–101T/068

TDC (CBCS) Odd Semester Exam., 2018

MATHEMATICS

(1st Semester)

Course No. : MTMHCC–101T

(Calculus)

Full Marks : 50

Pass Marks : 20

Time : 3 hours

The figures in the margin indicate full marks for the questions

UNIT—I

1. Answer any *two* of the following : 2×2=4

(a) If $y = \log(x+a)$, then find y_n.

(b) If $y = e^{ax}\sin bx$, then show that

$$y_2 - 2ay_1 + (a^2 + b^2)y = 0$$

(c) State Leibnitz rule on successive differentiation.

2. Answer *either* (a) and (b) *or* (c) and (d) :

(a) If $y = \sin mx$, then show that

$$\begin{vmatrix} y & y_1 & y_2 \\ y_3 & y_4 & y_5 \\ y_6 & y_7 & y_8 \end{vmatrix} = 0$$ 3

(b) If $y = e^{a \sin^{-1} x}$, then prove that

$$(1-x^2)y_{n+2} - (2n+1)xy_{n+1} - (n^2 + a^2)y_n = 0$$

Hence find $(y_n)_0$. 3

(c) If $x \sin\theta + y\cos\theta = a$ and $x\cos\theta - y\sin\theta = b$

then prove that

$$\frac{d^p x}{d\theta^p}\cdot\frac{d^q y}{d\theta^q} - \frac{d^q x}{d\theta^q}\cdot\frac{d^p y}{d\theta^p}$$

is constant. 3

(d) Prove that

$$\frac{d^n}{dx^n}(x^n \sin x) = n!(P\sin x + Q\cos x)$$

where $P = 1 - \binom{n}{2}\frac{x^2}{2} + \binom{n}{4}\frac{x^4}{4!} - \cdots$

$$Q = \binom{n}{1}x - \binom{n}{3}\frac{x^3}{3!} + \binom{n}{5}\frac{x^5}{5!} - \cdots$$ 3

UNIT—II

3. Answer any *two* of the following : 2×2=4

(a) Show that $\underset{x\to 0+}{\text{Lt}} (1+x)^{1/x} = e$.

(b) Define asymptote of a curve.

(c) Using the same set of rectangular axes, draw the graphs of the curves $y = \sin x$ and $y = \sin 2x$, $0 \le x \le 2\pi$.

4. Answer *either* (a) and (b) *or* (c) and (d) :

(a) Find : 1½×2=3

(i) $\underset{x\to 0}{\text{Lt}} \dfrac{(e^x - 1)\tan^2 x}{x^2}$

(ii) $\underset{x\to 0}{\text{Lt}} \left(\dfrac{1}{x^2} - \dfrac{1}{\sin^2 x}\right)$

(b) Examine the curve $y = x^3 - 3x + 3$ for concavity and points of inflection, if any. Hence trace the curve showing clearly the extreme points and points of reflection, if any. 3

(c) Evaluate : 1½×2=3

(i) $\underset{x\to 0}{\text{Lt}} (\cos x)^{\cot^2 x}$

(ii) $\underset{x\to 0}{\text{Lt}} \dfrac{xe^x - \log(1+x)}{x^2}$

(d) Find the asymptote of the curve

$$y = \frac{-8}{x^2 - 4}$$ 3

UNIT—III

5. Answer any *two* of the following : 2×2=4

(a) If $I_n = \int x^n \cos ax\, dx$

$J_n = \int x^n \sin ax\, dx$

then show that

$$aI_n = x^n \sin ax - nJ_{n-1}$$

(b) Evaluate $\int_0^{\pi/2} \cos^{10}\theta\, d\theta$.

(c) If $I_n = \int_0^{\pi/4} \tan^n \theta\, d\theta = \frac{1}{n-1} - I_{n-2}$, then find I_6.

6. Answer *either (a)* and *(b) or (c)* and *(d)* :

(a) From the reduction formula for $\int \cos^m x \cos nx\, dx$, obtain the value of $\int \cos^3 x \cos 5x\, dx$. 3

(b) If $I_{m,n} = \int_0^{\pi/2} \sin^m x \cos^n x\, dx$, $m, n \in \mathbb{N}$, then prove that $I_{m,n} = \frac{n-1}{m+n} I_{m,n-2}$. 3

(c) Obtain a reduction formula for $\int \sec^n x\, dx$. Hence find $\int \sec^6 x\, dx$. 3

(d) Prove that

$$\int_0^{\pi/2} \cos^n x\, dx = \frac{(n-1)(n-3)\cdots 4\cdot 2}{n(n-2)\cdots 5\cdot 3}, \text{ if } n \text{ is odd}$$
$$= \frac{(n-1)(n-3)\cdots 3\cdot 1}{n(n-2)\cdots 4\cdot 2}\frac{\pi}{2}, \text{ if } n \text{ is even}$$ 3

UNIT—IV

7. Answer any *two* of the following : 2×2=4

(a) Find by integration the length of $y = 3x$ from $x = 0$ to $x = 3$.

(b) Find the area of the region bounded by the curve $y^2 = x$ and the line $y = x$.

(c) Show that the surface area of a sphere of radius a is $4\pi a^2$.

8. Answer *either (a)* and *(b) or (c)* and *(d)* :

(a) Determine the length of the arc of the cycloid $x = a(\theta + \sin\theta)$, $y = a(1 - \cos\theta)$ measured from the vertex. 3

(b) Find the area of the smaller portion enclosed by the curves $x^2 + y^2 = 9$ and $y^2 = 8x$. 3

(c) Find the length of the arc of the curve $x = a\cos^3\theta$, $y = a\sin^3\theta$ lying in the 1st quadrant. 3

(d) Find the area of the surface of the solid generated by revolving the arc of the parabola $y^2 = 4ax$ bounded by the latus rectum about X-axis. 3

UNIT—V

9. Answer any *two* of the following : 2×2=4

(a) Prove that

$$\vec{a}\times\vec{b} = \{(\hat{i}\times\vec{a})\cdot\vec{b}\}\,\hat{i} + \{(\hat{j}\times\vec{a})\cdot\vec{b}\}\,\hat{j} + \{(\hat{k}\times\vec{a})\cdot\vec{b}\}\,\hat{k}$$

(b) Find the vector equation of the plane through the point $2\hat{i}+3\hat{j}-\hat{k}$ and perpendicular to the vector $3\hat{i}+4\hat{j}+7\hat{k}$.

(c) If $\vec{r} = 5t^2\hat{i} + t\hat{j} - t^3\hat{k}$ and $\vec{s} = \sin t\hat{i} - \cos t\hat{j}$

then find the values of $\frac{d}{dt}(\vec{r}\cdot\vec{s})$ and $\frac{d}{dt}(\vec{r}\times\vec{s})$.

10. Answer *either* (a) and (b) or (c) and (d) :

(a) Show that if $\vec{a}, \vec{b}, \vec{c}$ are non-coplanar, then $\vec{a}+\vec{b}, \vec{b}+\vec{c}, \vec{c}+\vec{a}$ are also non-coplanar. Is this true for $\vec{a}-\vec{b}, \vec{b}+\vec{c}, \vec{c}+\vec{a}$? 3

(b) Find the vector equation of the plane passing through a given point parallel to two given vectors. 3

(c) Show that $\vec{a}\times(\vec{b}\times\vec{c}) = (\vec{a}\times\vec{b})\times\vec{c}$ if and only if either $\vec{b}=\vec{0}$ or $\vec{c}$ is collinear with $\vec{a}$, or $\vec{b}$ is perpendicular to both $\vec{a}$ and $\vec{c}$. 3

(d) Prove that the necessary and sufficient condition for a vector $\vec{r} = \vec{f}(t)$ to have a constant direction is $\vec{f}\times\frac{d\vec{f}}{dt} = \vec{0}$. 3

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