**2023/TDC(CBCS)/EVEN/SEM/**
**MTMDSE–601T (A/B/C)/038**

**TDC (CBCS) Even Semester Exam., 2023**

# MATHEMATICS

**( 6th Semester )**

Course No. : MTMDSE–601T

*The figures in the margin indicate full marks for the questions*

Candidates have to answer from *either* Option—A *or* Option—B *or* Option—C

## OPTION—A

Course No. : MTMDSE–601T (A)

**( Complex Analysis )**

*Full Marks* : 70

*Pass Marks* : 28

*Time* : 3 hours

### SECTION—A

Answer any *twenty* of the following questions : 1×20=20

**1.** Express $\sqrt{3}-i$ in polar form.

**2.** Represent $1+i$ in Argand diagram.

3. If $z$ is a complex number such that $z+\bar{z}=0$, then what is the real part of $z$?

4. Write the triangle inequality for two complex numbers $z$ and $w$.

5. Express the following in the form $a+ib$ :

$$\frac{1+i}{1-i}$$

6. Let $f: \mathbb{C} \to \mathbb{C}$ where $\mathbb{C}$ is the set of complex numbers defined by

$$f(z) = z\bar{z} \quad \forall\ z \in \mathbb{C}$$

What is the range of $f$?

7. Write Cauchy-Riemann equations in polar form.

8. Evaluate :

$$\underset{z \to i}{\text{Lt}} \left( \frac{z-i}{z^2+1} \right)$$

9. Define continuity of a function at a point in $\mathbb{C}$.

10. Give example of a function that is continuous at every point in $\mathbb{C}$.





11. Evaluate :

$$\int_1^i z^3 dz$$

12. If $C$ is any simple closed curve, then what is the value of $\oint_C z dz$?

13. Give example of a simply connected region.

14. Give example of a multiply connected region.

15. If $f(z) = u(x, y) + iv(x, y)$, then write the relation between the complex and real line integrals.

16. Write Cauchy's integral formula.

17. Show that amp $z = -$amp $\bar{z}$.

18. Evaluate $\oint_C \frac{dz}{z-1}$, where $C$ is the circle $|z| = 2$.

19. State Cauchy's inequality.

20. Evaluate $\oint_C \frac{zdz}{(z-2)^2}$, where $C$ is the circle $|z| = 3$.

21. What is an entire function?

22. What do you mean by Jordan arc?

23. Write the Taylor series for $e^z$.

24. State the fundamental theorem of algebra.

25. Justify if $f(z) = \sin z$, $z \in \mathbb{C}$ is bounded.

SECTION—B

Answer any *five* of the following questions : 2×5=10

26. For any two complex numbers $z_1$ and $z_2$, show that

$$\overline{z_1 z_2} = \overline{z}_1 \cdot \overline{z}_2$$

27. Find the locus of the point $z$ satisfying

$$|z-1| = 2|z+1|$$

28. Evaluate :

$$\underset{z \to i}{\text{Lt}} \frac{z^2 - z - iz + i}{z^2 + 1}$$

29. Show that $u = 2x(1-y)$ is harmonic.





30. Evaluate :

$$\int_{3i}^{1-i} 4z\,dz$$

31. Evaluate $\int_C (12z^2 - 4iz)\,dz$ along the straight line joining (1, 1) and (2, 1).

32. Evaluate

$$\oint_C \frac{e^{2z}}{(z+1)^2}\,dz$$

where $C$ is the circle $|z| = 2$

33. Let $C$ be a simple closed curve enclosing the point $z = a$. What is the value of

$$\oint_C \frac{f(z)}{(z-a)^5}\,dz?$$

34. Find the zeroes of the polynomial

$$z^3 - 3z^2 + z - 3$$

35. State Taylor's theorem. How can you obtain Maclaurin's series from Taylor's series?

SECTION—C

Answer any *five* of the following questions : 8×5=40

**36.** *(a)* Describe the locus represented by

$$|z|+|z-4|=6$$

and obtain the Cartesian equation. 4

*(b)* Find the condition on

$$\frac{(z_3-z_1)(z_4-z_2)}{(z_3-z_2)(z_4-z_1)}$$

so that the points $z_1$, $z_2$, $z_3$, $z_4$ are concyclic. 4

**37.** *(a)* Describe the geometrical interpretation of

$$\arg\left(\frac{z-\alpha}{z-\beta}\right)$$ 4

*(b)* Find the expression for the area of a triangle with vertices $z_1$, $z_2$, $z_3$. 4

**38.** *(a)* Derive a necessary condition for a function to be analytic in a region $D \subseteq \mathbf{C}$. 4

*(b)* If $u = e^{-x}(x\sin y - y\cos y)$, then find $v$ such that $f(z) = u + iv$ is analytic. 4





**39.** *(a)* Evaluate : 3

$$\underset{z\to i}{\text{Lt}}\ \frac{z^2-2iz-1}{z^4+2z^2+1}$$

*(b)* Discuss the differentiability of the function $f:\mathbf{C}\to\mathbf{C}$ defined by

$$f(z)=|z|^2\quad \forall\, z\in \mathbf{C}$$

at all points in **C**. 5

**40.** *(a)* Evaluate

$$\int_C f(z)\,dz$$

where $f(z)=z^3$ and $C$ is the curve $\gamma(t)=t^2+it$ for $t\in[0,\pi]$. 4

*(b)* Justify whether $\int_C \text{Re}(z)\,dz$ is independent of the path joining 0 and $1+i$. 4

**41.** State and prove Cauchy-Goursat theorem for the case of a triangle. 2+6=8

**42.** *(a)* State and prove Morera's theorem. 5

*(b)* Evaluate

$$\oint_C \frac{\sin\pi z^2+\cos\pi z^2}{(z-1)(z-2)}\,dz$$

where $C$ is the circle $|z|=3$. 3

**43.** *(a)* If $f$ is analytic inside and on the circle $C$ with radius $r$ and centre $z = a$, and $|f(z)| \le M$ on $C$, then show that

$$|f^n(a)| \le \frac{M \cdot n!}{r^n}, \quad n = 0, 1, 2, \cdots$$ 5

*(b)* Evaluate

$$\oint_C \frac{e^{3z}}{(z - i\pi)^2} dz$$

where $C$ is the ellipse $|z-2| + |z+2| = 6$. 3

**44.** *(a)* State and prove Liouville's theorem. 5

*(b)* Derive the Taylor series for

$$f(z) = \ln(1+z)$$

about $z = 0$. 3

**45.** *(a)* Use the fundamental theorem of algebra to show that every polynomial of degree $n$ has exactly $n$ zeros. 4

*(b)* Expand $f(z) = \dfrac{1}{1+z}$ about $z = 1$ and determine the region of convergence. 4





OPTION—B

Course No. : MTMDSE–601T (B)

**( Linear Programming )**

*Full Marks* : 70

*Pass Marks* : 28

*Time* : 3 hours

SECTION—A

Answer any *twenty* of the following questions : 1×20=20

1. What is unbounded solution?

2. Find the extreme points of the following set :

$$A = \{(x_1, x_2) \mid |x_1| \le 1, |x_2| \le 1\}$$

3. Is $x_1 = 1$, $x_2 = \frac{1}{2}$, $x_3 = x_4 = x_5 = 0$ a basic solution of the following system?

$$x_1 + 2x_2 + x_3 + x_4 = 2$$

$$x_1 + 2x_2 + \frac{1}{2}x_3 + x_5 = 2$$

4. Express $\left(0, \frac{3}{2}\right)$, if possible, as a convex combination of (1, 1) and (–1, 2).

5. When are two hyperplanes parallel?

6. Under what conditions an LPP may have an alternative optimal solution?

7. What is degeneracy in LPP?

8. What is the significance of taking a large positive value of *M* in Big-*M* method?

9. In the two-phase method, when can we say that the LPP has no feasible solution?

10. State the fundamental theorem of linear programming model.

11. What is the advantage of dual simplex algorithm?

12. State the weak duality theorem.

13. What do you mean by a balanced transportation problem?

14. What are the disadvantages of North-West corner rule?





15. Why do we use Vogel's approximation method?

16. What is an unbalanced assignment problem?

17. Why can degeneracy arise in the solution of a transportation problem?

18. What is the indication that a transportation problem has multiple optimal solutions?

19. How would you explain the interpretation of the optimal solution of an unbounded TP?

20. How would you apply North-West corner rule to find initial feasible solution if the first-source, first-destination route is prohibited?

21. What do you mean by a 'finite game'?

22. What do you mean by pure and mixed strategy?

23. What is maximin principle?

24. What is saddle point?

25. Give one example of a non-zero-sum game.

SECTION—B

Answer any *five* of the following questions : 2×5=10

**26.** A hyperplane is given by the equation

$$3x_1 + 3x_2 + 4x_3 + 7x_4 = 8$$

Find in which half spaces the points (−6, 1, 7, 2) and (1, 2, −4, 1) lie.

**27.** Find all the basic solutions of the following system :

$$x_1 + 2x_2 + x_3 = 4$$
$$2x_1 + x_2 + 5x_3 = 5$$

**28.** Describe phase I of two-phase method for LPP.

**29.** Construct the initial simplex table for the following LPP :

Maximize $Z = x_1 + 2x_2 + 3x_3 - x_4$

subject to

$$x_1 + 2x_2 + 3x_3 = 15$$
$$2x_1 + x_2 + 5x_3 = 20$$
$$x_1 + 2x_2 + x_3 + x_4 = 10$$
$$x_1, x_2, x_3, x_4 \geq 0$$





**30.** Using one example, show that the dual of the dual of a given primal is the primal itself.

**31.** Determine an initial basic feasible solution to the following transportation problem using the North-West corner rule :

| | | Destination | | | | |
|---|---|---|---|---|---|---|
| | | $D_1$ | $D_2$ | $D_3$ | $D_4$ | *Availability* |
| | $S_1$ | 6 | 4 | 1 | 5 | 14 |
| *Source* | $S_2$ | 8 | 9 | 2 | 7 | 16 |
| | $S_3$ | 4 | 3 | 6 | 2 | 5 |
| *Requirement* | | 6 | 10 | 15 | 4 | |

**32.** Write a note on assignment problem.

**33.** Give the pictorial (schematic) presentation of MODI method.

**34.** Find the saddle point of the following pay-off matrix :

$$\begin{array}{cc} & B \\ & \begin{array}{ccc} I & II & III \end{array} \\ A \begin{array}{c} I \\ II \end{array} & \begin{bmatrix} 6 & 8 & 6 \\ 4 & 12 & 2 \end{bmatrix} \end{array}$$

**35.** Construct the corresponding LPP for the following pay-off matrix :

$$A\begin{matrix} & \\ I \\ II \\ III \end{matrix}\overset{\displaystyle B}{\begin{matrix} I & II & III \\ \end{matrix}}$$

| | | B I | B II | B III |
|---|---|---|---|---|
| A | I | 5 | 3 | 7 |
| A | II | 7 | 9 | 1 |
| A | III | 10 | 6 | 2 |

## SECTION—C

Answer any *five* of the following questions : 8×5=40

**36.** *(a)* Prove that the intersection of two convex sets is also a convex set. 3

*(b)* Solve graphically : 5

Maximize $Z = 6x_1 + 11x_2$

subject to

$$2x_1 + x_2 \le 104$$
$$x_1 + 2x_2 \le 76$$
$$x_1 \ge 0, \; x_2 \ge 0$$





**37.** *(a)* A firm manufactures two products $A$ and $B$ on which the profits earned per unit are ₹3 and ₹4 respectively. Each product is processed on two machines $M_1$ and $M_2$. Product $A$ requires 1 minute of processing on $M_1$ and 1 minute of processing on $M_2$. $M_1$ is not available for more than 7 hours 30 minutes while $M_2$ is available for 10 hours during any working day. Find the number of units of products $A$ and $B$ need to be manufactured to get maximum profit. Formulate the LP model. 5

*(b)* Write the following LPP in standard form : 3

Maximize $Z = 3x_1 + 2x_2$

subject to

$$-2x_1 + 3x_2 \le 9$$
$$x_1 - 5x_2 \ge -20$$
$$x_1, \; x_2 \ge 0$$

**38.** Solve the following by two-phase method : 8

Minimize $Z = x_1 + x_2$

subject to

$$2x_1 + x_2 \ge 4$$
$$x_1 + 7x_2 \ge 7$$
$$x_1, \; x_2 \ge 0$$

**39.** Solve the following LPP using simplex method : 8

Maximize $Z = 3x_1 + 5x_2 + 4x_3$

subject to

$2x_1 + 3x_2 \le 8$

$2x_2 + 5x_3 \le 10$

$3x_1 + 2x_2 + 4x_3 \le 15$

and $x_1, x_2, x_3 \ge 0$.

**40.** Using dual, solve the following LPP : 8

Maximize $Z_p = 3x_1 - 2x_2$

subject to

$x_1 \le 4$

$x_2 \le 6$

$x_1 + x_2 \le 5$

$-x_2 \le -1$

and $x_1, x_2 \ge 0$.

**41.** Determine an initial basic feasible solution to the following transportation problem using Vogel's approximation method : 8

| | | Destination | | | | |
|---|---|---|---|---|---|---|
| | | $D_1$ | $D_2$ | $D_3$ | $D_4$ | Availability |
| Source | $S_1$ | 1 | 2 | 1 | 4 | 20 |
| | $S_2$ | 3 | 3 | 2 | 1 | 40 |
| | $S_3$ | 4 | 2 | 5 | 9 | 20 |
| | $S_4$ | 5 | 3 | 6 | 10 | 20 |
| Requirement | | 20 | 40 | 30 | 10 | |





**42.** Solve the following balanced transportation problem : 8

| | | Destination | | | |
|---|---|---|---|---|---|
| | | $D_1$ | $D_2$ | $D_3$ | Availability |
| Source | $S_1$ | 8 | 7 | 3 | 60 |
| | $S_2$ | 3 | 8 | 9 | 70 |
| | $S_3$ | 11 | 3 | 5 | 80 |
| Requirement | | 50 | 80 | 80 | |

**43.** Solve the following minimal assignment problem : 8

| Man → / Job ↓ | 1 | 2 | 3 | 4 |
|---|---|---|---|---|
| I | 12 | 30 | 21 | 15 |
| II | 18 | 33 | 9 | 31 |
| III | 44 | 25 | 24 | 21 |
| IV | 23 | 30 | 28 | 14 |

**44.** Solve the game graphically whose pay-off matrix is 8

$$\begin{array}{cc} & B \\ & \begin{array}{cccc} I & II & III & IV \end{array} \\ A \begin{array}{c} I \\ II \end{array} & \begin{bmatrix} 1 & 3 & -3 & 7 \\ 2 & 5 & 4 & -6 \end{bmatrix} \end{array}$$

**45.** Solve the game using any appropriate method whose pay-off matrix is given by

| | | B | | | |
|---|---|---|---|---|---|
| | | I | II | III | IV |
| A | I | 3 | 2 | 4 | 0 |
| | II | 2 | 4 | 2 | 4 |
| | III | 4 | 2 | 4 | 0 |
| | IV | 0 | 4 | 0 | 8 |

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OPTION—C

Course No. : MTMDSE–601T (C)

**( Object-oriented Programming in C++ )**

*Full Marks* : 50
*Pass Marks* : 20

*Time* : 3 hours

SECTION—A

Answer any *fifteen* of the following questions : 1×15=15

**1.** What is object-oriented programming?

**2.** What is an object in C++?

**3.** Define member function.

**4.** Where does the name C++ come from?





**5.** How do you access the memory address of a variable?

**6.** What is wrong with the following code?

```
char c = 'w';
char p = c;
```

**7.** What name must a constructor have?

**8.** How many destructors can a class have?

**9.** Write a single C++ statement that prints "TOO MANY" if the variable count exceeds 100.

**10.** What is dangling pointer?

**11.** What is the difference between a class and a struct in C++?

**12.** When does a function need an include directive?

**13.** How many different types can the elements of an array have?

14. What happens if an array's initializer has more values than the size of the array?

15. What do you mean by operator overloading in C++?

16. What is virtual member function?

17. What is derived class in C++?

18. What is the function of scope resolution operator in C++?

19. Why cannot ** be overloaded as an exponential operator in C++?

20. What is an abstract base class?

SECTION—B

Answer any *five* of the following questions : 2×5=10

21. Define nested class. What are the restrictions on local class?

22. Write a short note on inheritance.





23. What are the disadvantages of OOP?

24. What is virtual function? Write the rules for defining a virtual function.

25. Explain the difference between copy constructor and assignment operator.

26. What is polymorphism?

27. What are the differences between passing a parameter by a value and by a reference?

28. What is the difference between 'static binding' and 'dynamic binding'?

29. What is the difference between the effects of the following two lines?

    Ratio y = x;
    Ratio; y = x;

30. What are the main differences between an array and a C++ vector?

## SECTION—C

Answer any *five* of the following questions : 5×5=25

31. Write a function that uses pointers to copy an array of double.

32. Write a program in C++ to overload plus (+) operator to carry out minus (–) operation using 4 components and hence find the value of

$$(4, 3, 8, 10) + (2, 8, 4, 12)$$

33. Write a program using OOP to find the roots of the following quadratic equation in $x$ :

$$ax^2 + bx + c = 0, \; a \neq 0$$

34. What is memory leak? How can virtual destructors plug a memory leak?

35. What is compile time polymorphism and how is it different from runtime polymorphism?

36. What are the various types of constructors in C++? Explain with examples.

37. Explain various data types used in C++.





38. Explain public and private access modifiers for C++ classes.

39. Explain pointer to derived class with example.

40. List the benefits of object-oriented programming.

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