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2021/TDC (CBCS)/EVEN/SEM/
MTMDSE-601T/128

TDC (CBCS) Even Semester Exam.,
September—2021

MATHEMATICS

(6th Semester)

Course No. : MTMDSE-601T

Full Marks : 70

Pass Marks : 28

Time : 3 hours

The figures in the margin indicate full marks for the questions

The Candidates have to answer *either* from Option—A *or* Option—B *or* Option—C

OPTION—A

Course No. : MTMDSE-601T (A)

(Linear Programming)

SECTION—A

Answer any *twenty* of the following questions : $1\times20=20$

1. Let x be the number of items produced in a factory per week. It is required to produce at least 200 items. Write the corresponding constraints.

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2. The profit earned by a company on products A, B and C are ₹ 5, ₹ 6 and ₹ 3 per unit respectively. The company sells x_1, x_2 and x_3 units of A, B and C respectively. Write the corresponding objective functions.

3. If $(2, 7) = x(1, 0) + y(1, 1)$ in $\mathbb{E}^2$, then find x and y.

4. Write a basis of the three-dimensional Euclidean space.

5. Define line segment in $\mathbb{E}^n$.

6. Define hyperplane in $\mathbb{E}^n$.

7. Give an example of a set in $\mathbb{E}^2$ that is not convex.

8. Define convex polyhedron.

9. To solve an LPP using simplex method, what should be the nature of the objective function?

10. What type of constraints should an LPP have in order to be solvable using simplex method?

(Continued

11. Name two methods for solving an LPP involving artificial variables.

12. While solving an LPP by Big-M method, what can you conclude in the situation where optimality conditions are satisfied but at least one artificial variable is present in the basis at positive level?

13. Find an initial basic feasible solution of the following LPP :

Maximize $Z = 3x + 5y$

subject to

$$2x - 3y \le 5$$
$$7x + 4y \le 8$$
$$x, y \ge 0$$

14. How many artificial variables are required for solving an LPP having two $\ge$ constraints with positive RHS?

15. In Big-M method, what is the cost assigned to each artificial variable?

16. What is the auxiliary objective function in two-phase method?

17. If there are 5 variables in the primal, how many constraints will be there in the dual?

(Turn Over)

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18. If the 3rd variable in the primal is unrestricted in sign, what can you say about the 3rd constraint in the dual?

19. State the necessary and sufficient condition for a transportation problem to have a feasible solution.

20. Write the expression for the objective function of a general transportation problem.

21. When is a transportation problem said to be unbalanced?

22. Which cell receives the first allocation in the north-west corner rule?

23. How is penalty calculated in Vogel's approximation method?

24. How will you convert an unbalanced transportation problem where total supply is more than the total demand to a balanced one?

25. In optimality test for transportation problem, what is the relation between C_{ij}, u_i and v_j for occupied cell? Here the symbols have their usual meanings.

26. Write the expression for cell evaluation for empty cells in performing optimality test for transportation problem.

27. Write the condition for the solution of transportation problem to be optimal and unique.

28. When is the initial solution to a transportation problem said to be degenerate?

29. Write the explicit form of the allocation x_{ij} in an assignment problem.

30. Name a method to solve an assignment problem.

31. In an assignment problem, how many jobs can be assigned to a person?

32. When is an assignment problem said to be unbalanced?

33. What is payoff matrix?

34. What is finite game?

35. What is two persons zero-sum game?

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36. State the minimax principle.

37. What is pure strategy?

38. What is mixed strategy?

39. When is a game said to have a saddle point?

40. What is a symmetric game?

SECTION—B

Answer any *five* of the following questions : 2×5=10

41. Plot the feasible region for an LPP with the following constraints :

$$2x+3y \le 6$$
$$x-y \le 1$$
$$x \ge 1,\ y \ge 0$$

42. Justify if the set

$$S = \{(x, y) \in \mathbb{E}^2 \mid x^2 + y^2 = 1\}$$

is a convex set.

43. Construct the coefficient matrix for an LPP with the following constraints after introducing slack variables, if necessary :

$$x+y-3z \le 8$$
$$2x+7y \le 9$$
$$x,\ y,\ z \ge 0$$

44. Write the subsequent steps to be followed in Big–M method once the optimality criterion is satisfied at any iteration.

45. Write the dual of the LPP :

Maximize $Z = 3x_1 + 4x_2$

subject to the constraints

$$x-7y \le 3$$
$$2x+3y \le 6$$
$$x+y \ge 2$$
$$x,\ y \ge 0$$

46. Find an initial basic feasible solution of the following transportation problem :

		Destinations			
		D_1	D_2	D_3	
Sources	S_1	3	10	2	15
	S_2	1	5	7	20
	S_3	8	3	2	25
		20	18	22	

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47. Check if the following solution to the transportation problem is optimal :

	M_1	M_2	M_3	M_4	
F_1	1	2	4	5	20
F_2	6	4	2	3	20
F_3	5	4	8	2	30
	15	25	15	15	

F_1 to M_1—15 units, F_1 to M_2—5 units, F_2 to M_2—20 units, F_3 to M_3—15 units, F_3 to M_4—15 units

48. Write the starting cost matrix for the following assignment problem :

		Machines			
		I	*II*	*III*	*IV*
Jobs	*A*	8	26	17	11
	B	13	28	4	26
	C	38	19	18	15
	D	19	26	24	10

49. Two boys *A* and *B* simultaneously draw either one ball or two balls which they have in their bags. If the number of balls drawn by *B* is the same as that drawn by *A*, then *A* wins and gets one rupee from *B*. On the other hand, if the numbers are not the same, then *B* wins and gets one rupee from *A*. Construct the payoff matrix for this game.

50. Find saddle point, if any, for the game whose payoff matrix is

		B			
		B_1	B_2	B_3	B_4
A	A_1	1	7	3	4
	A_2	5	6	4	5
	A_3	7	2	0	3

SECTION—C

Answer any *five* of the following questions : 8×5=40

51. *(a)* A company manufactures two types of lamps *A* and *B*. Both the lamps go through two technicians, first a cutter and second a finisher. Lamp *A* requires 2 hours of cutter's time and 3 hours of finisher's time. Lamp *B* requires 1 hour of cutter's time and 2 hours of finisher's time. The cutter has 120 hours and the finisher has at most 100 hours of duty per month. Profit on lamp *A* is ₹ 6 per unit and on lamp *B* is ₹ 3 per unit. The company wants to produce at least 50 units of lamp *A* per month. Formulate this problem as an LPP so as to maximize the profit. 3

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(b) Solve the following graphically : 5

$$\text{Minimize} \quad Z = 3x + 5y$$

subject to

$$5x + 3y \geq 15$$
$$-x + y \leq 3$$
$$x \leq 4$$
$$y \geq 3$$
$$x, y \geq 0$$

52. (a) Find all the basic solutions of

$$x - y + 2z = 1$$
$$2x - 2y + 3z = 2$$

4

(b) Show that arbitrary intersection of convex sets in $\mathbb{E}^n$ is convex. 4

53. (a) Write the following LPP in standard form, using slack or surplus whereas necessary : 3

$$\text{Maximize} \quad Z = x_1 + 2x_2 - x_3$$

subject to

$$x_1 - 2x_2 \geq 3$$
$$2x_1 + 4x_2 + x_3 \leq 8$$
$$x_1 + 7x_3 \leq 5$$
$$x_1, x_2, x_3 \geq 0$$

(b) Solve the following by simplex method : 5

$$\text{Maximize} \quad Z = 3x_1 + x_2 + 3x_3$$

subject to

$$2x_1 + x_2 + x_3 \leq 2$$
$$x_1 + 2x_2 + 3x_3 \leq 5$$
$$2x_1 + 2x_2 + x_3 \leq 6$$
$$x_1, x_2, x_3 \geq 0$$

54. (a) Write the steps involved in solving an LPP using two-phase method. 3

(b) Solve the following LPP : 5

$$\text{Maximize} \quad Z = -2x_1 - x_2$$

subject to

$$3x_1 + x_2 = 3$$
$$4x_1 + 3x_2 \geq 6$$
$$x_1 + 2x_2 \leq 4$$
$$x_1, x_2 \geq 0$$

55. (a) Write the dual of the problem

$$\text{Minimize} \quad Z = x_1 - x_2 + 7x_3$$

subject to the constraints

$$3x_1 - 7x_2 \geq 5$$
$$x_1 + x_2 + x_3 \leq 7$$
$$4x_1 - 5x_2 + 4x_3 \leq 8$$
$$x_1, x_2 \geq 0$$

and x_3 is unrestricted in sign. 3

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(b) Find an initial basic feasible solution of the following transportation problem using matrix minima method and check if the solution is optimal :

Markets

		M_1	M_2	M_3	
Sources	S_1	4	8	15	30
	S_2	7	3	5	40
	S_3	4	7	1	20
	S_4	2	2	5	10
		30	40	30	

56. *(a)* Write the steps to be adopted in order to solve an unbalanced transportation problem. 3

(b) Find an initial basic feasible solution of the following transportation problem using Vogel's approximation method : 5

	D_1	D_2	D_3	
S_1	2	7	4	5
S_2	3	3	1	8
S_3	5	4	7	7
S_4	1	6	2	14
	7	9	18	

57. *(a)* Find an optimal solution of the following transportation problem : 6

	D_1	D_2	D_3	D_4	
O_1	10	7	3	6	3
O_2	1	6	8	3	5
O_3	7	4	5	3	7
	3	2	6	4	

(b) Write a brief note on the resolution of degeneracy in a transportation problem. 2

58. *(a)* Solve the assignment problem : 5

	I	*II*	*III*	*IV*
A	8	26	17	11
B	13	28	4	26
C	38	19	18	15
D	19	26	24	10

(b) Write the steps involved in solving an assignment problem by Hungarian method. 3

59. *(a)* Solve the following game whose payoff matrix is given by

	I	*II*	*III*	*IV*
I	–5	3	1	20
II	5	5	4	6
III	–4	–2	0	–5

Show that the game is strictly determinable. 4

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(b) For what value of a, the game with the following payoff matrix is strictly determinable :

	I	II	III
I	a	5	2
II	−1	a	−8
III	−2	3	a

60. *(a)* Solve the game with the following payoff matrix :

1	3
4	2

(b) Solve graphically :

	B_1	B_2	B_3
A_1	1	3	11
A_2	8	5	2

OPTION—B

(For Pass Students)

Course No. : MTMDSE–601T (B)

(Complex Analysis)

SECTION—A

Answer any *twenty* of the following questions : 1×20=20

1. Write the equation of a circle with centre at (−2, 1) and radius 4.

2. Express $-5+5i$ in polar form.

3. Show that $\overline{(\overline{z})} = z$.

4. In the set of complex numbers, write the multiplicative identity.

5. Find the principal argument of $-7-5i$.

6. Give a geometrical interpretation of

$$|z_1 + z_2| \le |z_1| + |z_2|$$

7. Prove that

$$e^{i\theta} = e^{i(\theta + 2k\pi)},\ k = 0, \pm 1, \pm 2, \cdots$$

8. What is the area of a parallelogram having sides z_1 and z_2?

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9. Define limit of a function $f(z)$ at z_0.

10. For what values of z

$$f(z) = \frac{z}{z^2+1}$$

is continuous?

11. Define singularity of $f(z)$.

12. Define analytic function.

13. Give an example of a function which is continuous at a point but is not analytic at that point.

14. Name the singularity of the function

$$\sin^{-1}\left(\frac{1}{z}\right)$$

at $z=0$.

15. Evaluate :

$$\lim_{z\to 0} \frac{1-\cos z}{z^2}$$

16. Evaluate $\frac{dw}{dz}$, where

$$w = \frac{1+z}{1-z}$$

(Continued)

17. Define simply connected region.

18. If a and b are any two points in the region R and $F'(z) = f(z)$, then what is the value of

$$\int_a^b f(z)\, dz \,?$$

19. Evaluate $\int_C \bar{z}\, dz$, from $z=0$ to $z=4+2i$ along the curve C given by $z=t^2+it$.

20. Define contour integral.

21. Evaluate $\int_C (z-z_0)$, where C is any simple closed curve and z_0 is a constant.

22. Find

$$\int_0^1 ze^{2z} dz$$

23. Evaluate $\oint_C \frac{dz}{z-a}$, where C is any simple closed curve and $z=a$ is outside C.

24. Evaluate

$$\int_i^{2-i} (3xy+iy^2)\, dz$$

along the straight line joining $z=i$ and $z=2-i$.

(Turn Over)

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25. State Cauchy's integral formula for $f'(a)$.

26. Evaluate

$$\frac{1}{2\pi i}\int_C \frac{e^z}{z-2}dz$$

if C is the circle $|z|=3$.

27. State Cauchy's inequality.

28. Evaluate $\int_C \frac{z+2}{z}dz$, where C is the circle $|z|=1$.

29. State Cauchy's integral formula for all positive integral values of n.

30. Evaluate $\int_C \frac{e^{iz}}{z^3}dz$, where C is the circle $|z|=2$.

31. Find the value of

$$\int_C \frac{\sin^6 z}{z-\frac{\pi}{6}}dz$$

if C is the circle $|z|=1$.

32. Evaluate $\int_C \frac{e^z}{(z+1)^2}dz$, where C is the circle $|z|=3$.

33. Define an integral function.

34. What is the region of convergence in the Taylor series of the function $f(z)=\sin z$?

35. Determine the zeroes of

$$f(z)=z^4-z^2-2z+2$$

36. State Maclaurin series.

37. State the condition for the validity of the expansion of a function as a Taylor series.

38. Write the expansion of e^{-z} as Taylor series within the region of convergence.

39. Write the condition under which $\tan^{-1} z$ can be expanded as a Taylor series.

40. What is the Taylor series expansion of

$$\log\left(\frac{1+z}{1-z}\right)$$

at $z=0$?

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SECTION—B

Answer any *five* of the following questions : 2×5=1

41. Find real numbers x and y such that

$$3x + 2iy - ix + 5y = 7 + 5i$$

42. If z_1 and z_2 are two complex numbers, show that

$$|z_1 + z_2|^2 + |z_1 - z_2|^2 = 2(|z_1|^2 + |z_2|^2)$$

43. If

$$f(z) = \begin{cases} z^2, & z \neq i \\ 0, & z = i \end{cases}$$

show that $f(z)$ is not continuous at $z = i$.

44. Prove that $u = y^3 - 3x^2 y$ is a harmonic function.

45. Define the definite integral of $f(z)$ from a to b along a rectifiable arc L.

46. State Cauchy-Goursat theorem.

47. Evaluate

$$\int_C \frac{1}{z(z-2)}\, dz$$

where C is the circle $|z| = 1$.

48. State Cauchy's integral formula.

49. Expand

$$f(z) = \frac{1}{(z+1)(z+3)}$$

in a series valid for the region $|z| < 1$.

50. State Taylor's theorem of an analytic function $f(z)$.

SECTION—C

Answer any *five* of the following questions : 8×5=40

51. (a) If z_1, z_2 and z_3 are the vertices of an isosceles triangle right angled at the vertex z_2, prove that

$$z_1^2 + 2z_2^2 + z_3^2 = 2z_2(z_1 + z_3)$$ 4

(b) Give geometrical interpretation of

$$\arg\left(\frac{z - \alpha}{z - \beta}\right)$$ 4

52. (a) Find the equation of the circle having the line joining z_1 and z_2 as diameter. 4

(b) If z_1 and z_2 are two non-zero complex numbers such that $|z_1 + z_2| = |z_1| + |z_2|$, then find $\arg(z_1) - \arg(z_2)$. 4

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53. *(a)* Derive Cauchy-Riemann partial differential equation.

(b) Show that an analytic function with constant modulus is constant.

54. *(a)* If $u = e^x(x\cos y - y\sin y)$, find the analytic function $u + iv$.

(b) Prove that if

$$u = x^2 - y^2,\ v = -y/(x^2 + y^2)$$

both u and v satisfy Laplace's equation but $u + iv$ is not an analytic function of z.

55. *(a)* Evaluate

$$I = \int_{(0,\ 1)}^{(2,\ 5)} (3x + y)\,dx + (2y - x)\,dy$$

along—

(i) the curve $y = x^2 + 1$

(ii) the line joining (0, 1) and (2, 5) 2+2=

(b) Evaluate $\int (\bar{z})^2 dz$ around the circle $|z - 1| = 1$.

56. *(a)* Prove that a line integral of $f(z)$ over an arc L depends only on the end points of L.

(Continued

(b) If a function $f(z)$ is continuous on a contour L of length l and if M be the upper bound of $|f(z)|$ on L, i.e., $|f(z)| \leq M$ on L, then prove that

$$\left| \int_L f(z)\,dz \right| \leq ML$$ 4

57. *(a)* Let $f(z)$ be analytic function within and on the boundary C of a simply connected region D and let z_0 be any point within C, then show that

$$f'(z_0) = \frac{1}{2\pi i} \int_C \frac{f(z)}{(z - z_0)^2} dz$$ 5

(b) Evaluate

$$\int_C \frac{e^{ax}}{z^2 + 1} dz$$

where C is the circle $|z| = 2$, $x = \text{Re}(z)$. 3

58. *(a)* Evaluate

$$\int_C \frac{dz}{z^2 + 2z + 2}$$

where C is the square having vertices at (0, 0), (−2, 0), (−2, −2) and (0, 2) oriented in anticlockwise direction. 3

(b) State and prove Morera's theorem. 5

(Turn Over)

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59. (a) State and prove Liouville's theorem. 4

(b) Obtain the Taylor's series for

$$f(z)=\frac{z^2-1}{(z+2)(z+3)}$$

in the region $2<|z|<3$. 4

60. (a) State and prove fundamental theorem of algebra. 5

(b) Expand $\log(1+z)$ in a Taylor's series about $z=0$ and determine the region of convergence for the resulting series. 3

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OPTION—C

Course No. : MTMDSE–601T (C)

(Object-Oriented Programming in C++)

SECTION—A

Answer any *twenty* of the following as directed :
1×20=20

1. Which header file is used at the beginning of C++ program?

2. Write the name of the common function which is used in the beginning and end of both C and C++ program.

3. What is the meaning of iostream?

4. What is the full form of 'cin'?

5. What is the full form of 'cout'?

6. C++ is a _____ of C language.
(Fill in the blank)

7. Can C and C++ program compile in a same compiler?

8. What does the modulus operator '%' do?

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9. By how much does the increment operator increase the value of a variable?

10. What is the key to reusability in object-oriented programming?

11. What is inheritance?

12. What is polymorphism?

13. Define data abstraction.

14. How is OOP a natural way of programming?

15. What are arrays?

16. What is an abstract class?

17. What is the use of delete operator?

18. What is the purpose of an ADT?

19. What is a base class?

20. What is a derived class?

21. What is a constructor?

22. State the use of static data member of a class.

23. What is virtual base class?

24. State the use of keyword 'private'.

25. What is the use of 'public' keyword?

26. What is a friend function in C++?

27. Which keyword is used to represent a friend function?

28. What symbol is used to represent scope resolution operator?

29. What is static class member?

30. What is class templete?

31. What is operator overloading?

32. Write the syntax of an operator function.

33. How many arguments are required in the definition of an overloaded unary operator?

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34. Operator functions never return a value.

(Write True or False)

35. The overloaded operator must have at least one operand that is user-defined type.

(Write True or False)

36. Friend functions cannot be used to overload operators.

(Write True or False)

37. A constructor can be used to convert a basic type to a class type data.

(Write True or False)

38. Operator overloading works when applied to class objects only.

(Write True or False)

39. What is the utility of using 'endl' in C++ program?

40. What is the meaning of '&&' symbol in C++ programming?

SECTION—B

Answer any *five* of the following questions : 2×5=10

41. If $a = 100$ and $b = 4$, then determine the result of the following :

(a) $a += b$

(b) $a\% = b$

42. What is the need of object-oriented programming paradigm?

43. Define encapsulation.

44. Write any two advantages of inheritance.

45. Write down the syntax and example to create a class.

46. Define reference variable. Give its syntax.

47. What is the need of abstract class in C++?

48. What is the need of overloading operators and functions?

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49. Write any two advantages of C++ over C.

50. What is the difference between 'call by value' and 'call by reference'?

SECTION—C

Answer any *five* of the following questions : 8×5=40

51. Explain the structure of C++ program with example. 8

52. (a) What is object-oriented programming? How is it different from the procedure-oriented programming? 6

(b) What are the unique advantages of an object-oriented paradigm? 2

53. What are the different ways to define member functions of a class? What is the role of scope resolution operator in the definition of member function? 8

54. Write a C++ program to read two numbers from the keyboard and display the larger value on the screen. 8

55. Write a program to add two complex numbers using object as arguments. 8

56. Write a C++ program to calculate the roots of a quadratic equation by initializing the object using default constructor. 8

57. A friend function cannot be used to overload the assignment operator =. Explain why. When is a friend function compulsory? Give an example. 5+1+2=8

58. Write a C++ program to convert temperature from Fahrenheit to Centigrade and vice versa. 8

59. Write down the rules for overloading operators. 8

60. Write a C++ program to multiply the private members of two classes using a friend function. 8

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