TDC (CBCS) Odd Semester Exam., 2022

MATHEMATICS

(5th Semester)

Course No. : MTMDSE-502T

Full Marks : 70
Pass Marks : 28

Time : 3 hours

The figures in the margin indicate full marks for the questions

Candidates have to answer from *either* Option—A *or* Option—B

OPTION—A

Course No. : MTMDSE-502T (A)

(Analytical Geometry)

UNIT—I

1. Answer any *four* of the following questions : 1×4=4

(a) If the origin be shifted to (1, 5), find the new equation of the line $3x+4y=5$.

(b) If $ax^2 + 2hxy + by^2 = 0$ represents a pair of straight lines, then write the condition that they are real and coincident.

(c) Show that the equation

$$x^2 - 5xy + 6y^2 = 0$$

represents a pair of straight lines passing through origin.

(d) Find the equation of bisection of the angles between pair of lines

$$x^2 - 7xy + 12y^2 = 0$$

(e) Write the angle between the lines represented by

$$ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$$

2. Answer any *one* of the following questions : 2

(a) At what point the origin be shifted, if the coordinates of a point (4, 5) become (3, 7)?

(b) Find the equations of the lines whose joint equation is

$$x^2 + 2xy\sec\theta + y^2 = 0$$

3. Answer any *one* of the following questions : 8

(a) (i) What angle must the axes be turned to remove the term xy from

$$x^2 + 2\sqrt{3}xy - y^2 = 4?$$ 4

(ii) Prove that the product of perpendiculars falls from the point (x_1, y_1) upon the pair of lines $ax^2 + 2hxy + by^2 = 0$ is

$$\frac{ax_1^2 + 2hx_1y_1 + by_1^2}{\sqrt{(a-b)^2 + 4h^2}}$$ 4

(b) (i) Find the angle between the pair of lines represented by

$$ax^2 + 2hxy + by^2 = 0$$ 4

(ii) Prove that the equation of the bisector of the angles between the lines represented by

$$ax^2 + 2hxy + by^2 = 0$$

is

$$\frac{x^2 - y^2}{a-b} = \frac{xy}{h}$$ 4

UNIT—II

4. Answer any *four* of the following questions : 1×4=4

(a) Find the radical axis of the two circles

$$x^2 + y^2 + 4x - 2y + 9 = 0$$

and $$x^2 + y^2 + 2x + 3y - 5 = 0$$

(b) Show that the circles

$$x^2 + y^2 - 2bx + c = 0$$

and $$x^2 + y^2 + 2ay - c = 0$$

cut each other orthogonally.

(c) Find the equation of the tangent at the point (1, 4) to the ellipse $3x^2 + 7y^2 = 115$.

(d) Find the length of the tangent to the circle $x^2 + y^2 = 4$ from the point (2, 3).

(e) Write down the condition for a straight line $y = mx + c$ to be tangent to the hyperbola

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$$

5. Answer any *one* of the following questions : 2

(a) Find the radical centre of the set of circles

$$x^2 + y^2 + x + 2y + 3 = 0$$
$$x^2 + y^2 + 2x + 4y + 5 = 0$$
$$x^2 + y^2 - 7x - 8y - 9 = 0$$

(b) Prove that the straight line $lx + my + n = 0$ touches the parabola $y^2 = 4ax$, if $\ln = am^2$.

6. Answer any *one* of the following questions : 8

(a) (i) Prove that four normals can be drawn to a hyperbola

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$$ 4

(ii) If any tangent to the ellipse

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

makes intercepts of lengths h and k on the axes, then prove that

$$\frac{a^2}{h^2} + \frac{b^2}{k^2} = 1$$ 4

(b) (i) Prove that two circles which pass through two points $(0, a)$ and $(0, -a)$ and touch the straight line $y = mx + c$ will cut orthogonally, if

$$c^2 = a^2(2 + m^2).$$ 5

(ii) If the normal at the point $(at_1^2, 2at_1)$ on the parabola $y^2 = 4ax$ meets it again at the point $(at^2, 2at)$, then show that

$$t = -t_1 - \frac{2}{t_1}.$$ 3

UNIT—III

7. Answer any *four* of the following questions : 1×4=4

(a) Find the pole of the straight line $2x - 5y = 4$ with respect to the parabola $y^2 = 8x$.

(b) Write down polar of the point (x_1, y_1) with respect to the circle

$$x^2 + y^2 + 2gx + 2fy + c = 0$$

(c) Write down one property of pole and polar.

(d) If (r_1, θ_1) and (r_2, θ_2) be the polar coordinates of the two points P and Q, then find the distance PQ.

(e) Write down the equation of the tangent to the conic

$$\frac{l}{r} = 1 + e\cos\theta$$

at the point whose vectorial angle is α.

8. Answer any *one* of the following questions : 2

(a) Show that the locus of the poles of tangents to the parabola $y^2 = 4ax$ with respect to the parabola $y^2 = 4bx$ is the parabola

$$y^2 = \frac{4b^2}{a}x.$$

(b) Find the polar equation of the straight line joining two points $(1, \pi/2)$ and $(2, \pi)$.

9. Answer any *one* of the following questions : 8

(a) (i) Prove that the locus of the poles of the normal chords of the parabola $y^2 = 4ax$ is the curve

$$y^2(x+2a)+4a^3=0$$ 4

(ii) Find the equation of the tangent at the point (r_1, θ_1) of the conic

$$\frac{l}{r} = 1 + e\cos\theta$$

where $2l$ = length of latus rectum. 4

(b) (i) Find the equation of the polar of the point (x_1, y_1) with respect to the hyperbola

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$$ 4

(ii) Prove that the length of the focal chord of the conic

$$\frac{l}{r} = 1 - e\cos\theta$$

which is inclined to the axis at an angle α is

$$\frac{2l}{1 - e^2\cos^2\alpha}$$ 4

UNIT—IV

10. Answer any *four* of the following questions : 1×4=4

(a) Define non-coplanar lines.

(b) Show that the planes

$$2x-3y+6z=1$$
$$3x+6y-z=2$$
$$x+y+z=1$$

intersect at a point.

(c) Write down the condition that

$$ax^2+by^2+cz^2+2fyz+2gzx+2hxy+2ux+2vy+2wz+d=0$$

will represent a sphere.

(d) Find the length of the tangent to a sphere

$$x^2+y^2+z^2+2x+2y+2z+2=0$$

from the point (1, 2, 1).

(e) Find the equation of the sphere joining $P(2, -3, 4)$ and $Q(-5, 6, -7)$ as diameter.

11. Answer any *one* of the following questions : 2

(a) Find the length of shortest distance between the lines

$$\frac{x-1}{2}=\frac{y-2}{3}=\frac{z-3}{4}$$

and $$\frac{x-2}{3}=\frac{y-4}{4}=\frac{z-5}{5}$$

(b) Find the equation of the sphere passing through the four points $(1, -1, -1)$, $(3, 3, 1)$, $(-2, 0, 5)$ and $(-1, 4, 4)$.

12. Answer any *one* of the following questions : 8

(a) (i) Find the shortest distance between the straight lines

$$\frac{x-3}{-3}=\frac{y-8}{1}=\frac{z-3}{-1}$$

and $$\frac{x+3}{3}=\frac{y+7}{-2}=\frac{z-6}{-4}$$

and the equations of the line of shortest distance. 4

(ii) Find the centre and the radius of the circle

$$x^2+y^2+z^2=25$$
$$x+2y+2z+9=0$$ 4

(b) (i) Find the equation of the sphere which passes through the points $(1, 0, 0)$, $(0, 1, 0)$, $(0, 0, 1)$ and which touches the plane $2x+2y-z=15$. 5

(ii) Show that the shortest distance between axis of z and the line

$$ax+by+cz+d=0$$

and $$a_1x+b_1y+c_1z+d_1=0$$

is

$$\frac{dc_1-d_1c}{\sqrt{(ac_1-a_1c)^2+(bc_1-b_1c)^2}}$$ 3

UNIT—V

13. Answer any *four* of the following as directed : 1×4=4

(a) Write down the general equation to the cone of the second degree which passes through the axes.

(b) If l, m, n be the direction ratios of the generator OQ passing through the origin O, then write the equation of OQ.

(c) What is generator of a cylinder?

(d) What is enveloping cylinder?

(e) The second degree homogeneous equation represents ____ with vertex at origin.
(Fill in the blank)

14. Answer any *one* of the following questions : 2

(a) Find the equation of the right circular cone whose vertex is the origin, axis is the

$$\frac{x}{2}=\frac{y}{3}=\frac{z}{4}$$

and semi-vertical angle is 45°.

(b) Find the equation of the right circular cylinder of radius 3 and whose axis is

$$\frac{x-1}{2}=\frac{y-2}{-3}=\frac{z-3}{6}$$

15. Answer any *one* of the following questions : 8

(a) *(i)* Find the equation of the right circular cone with vertex at (3, 2, 1), semivertical angle 30° and axis

$$\frac{x-3}{1}=\frac{y-2}{4}=\frac{z-1}{3}$$ 4

(ii) Find the equation of a right circular cylinder of radius r and axis is

$$\frac{x-\alpha}{l}=\frac{y-\beta}{m}=\frac{z-\gamma}{n}$$ 4

(b) *(i)* Show that the equation of the right circular cone whose vertex is the point (0, 0, 3) and whose guiding curve is $x^2+y^2=4,\ z=0$ is

$$9(x^2+y^2)=4(z-3)^2$$ 4

(ii) Find the equation of a cylinder whose generating lines have direction cosines (l, m, n) and which passes through the circle

$$x^2+z^2=a^2,\ y=0$$ 4

OPTION—B

Course No. : MTMDSE-502T (B)

(Probability and Statistics)

UNIT—I

1. Answer any *four* of the following as directed : 1×4=4

(a) Define discrete sample space.

(b) If $\sum_{i=1}^{n} P(A_i) = 1$, then events are mutually exclusive.

(Write True or False)

(c) When two dice are thrown, find the probability that the difference of the points on the dice is 2 or 3.

(d) Define moment generating function.

(e) Define characteristic function of a random variable Y.

2. Answer any *one* of the following questions : 2

(a) A bag contains 4 white and 2 black balls and another bag contains 3 white and 5 black balls. If one ball is drawn from each bag, find the probability that one is white and the other is black.

(b) Find the probability distribution of the number of heads when three coins are tossed.

3. Answer any *one* of the following questions : 8

(a) (i) Cards are drawn at random, one at a time, from a well-shuffled pack of 52 playing cards until 2 aces are obtained for the first time. If N is the number of cards required to be drawn, then show that

$$P(\{N = n\}) = \frac{(n-1)(52-n)(51-n)}{50 \times 49 \times 17 \times 13}$$

where $2 \le n \le 50$. 4

(ii) From an urn containing 3 red and 2 white balls, a man is to draw two balls at random without replacement being promised ₹ 20 for each red ball he draws and ₹ 10 for each white ball. Find his expectation. 4

(b) (i) Two bad eggs are mixed accidentally with 10 good ones. Find the probability distribution of the number of bad eggs in 3 drawn at random without replacement, from this lot. 4

(ii) If

$$\frac{(1-3p)}{2}, \frac{(1+4p)}{3}, \frac{(1+p)}{6}$$

are the probabilities of three mutually exclusive and exhaustive events, then prove that the set of all values of p is in $\left[-\frac{1}{4}, \frac{1}{3}\right]$. 4

UNIT—II

4. Answer any *four* of the following as directed : 1×4=4

(a) What is Bernoulli distribution?

(b) Given that X has Poisson distribution with variance 0·5, calculate $P(X=3)$.

(c) If X has a binomial distribution with parameters p and n, then $\text{var}(X)=$ _____.

(Fill in the blank)

(d) Define uniform distribution.

(e) Define exponential distribution.

5. Answer any *one* of the following questions : 2

(a) Determine the binomial distribution whose mean is 9 and standard deviation is $\frac{3}{2}$.

(b) If X follows a normal distribution with mean 100 and variance 25, find $P(|X-100|\leq 5)$ given that

$$\frac{1}{\sqrt{2\pi}}\int_{-\infty}^{1} e^{-x^2/2}dx = 0\cdot 8413447$$

6. Answer any *one* of the following questions : 8

(a) (i) If X has a binomial distribution with parameters p and n, then prove that $E(X)=np$ and $\text{var}(X)=npq$. 5

(ii) Let X be a Poisson distributed random variable with parameter μ. Then find $\text{var}(X)$. 3

(b) If X is a normal random variable, i.e., $X \sim N(\mu, \sigma^2)$, then find $E(X)$ and $\text{var}(X)$.

UNIT—III

7. Answer any *four* of the following questions : 1×4=4

(a) What is two-dimensional distribution function?

(b) Define conditional expectation in case of discrete distribution.

(c) Write one property of joint distribution function.

(d) Define expectation of two-dimensional random variable.

(e) If (X, Y) be a two-dimensional random variable, then write the formula for $E(X+Y)$.

8. Answer any *one* of the following questions : 2

(a) Determine the value of the constant k, such that the function $f(x)$ defined by

$$f(x) = \begin{cases} kx(1-x), & 0 < x < 1 \\ 0 \quad , & \text{elsewhere} \end{cases}$$

is a probability density function of some distribution.

(b) The random variables X and Y have the joint density function

$$f(x, y) = \begin{cases} 6(1-x-y), & x > 0,\ y > 0,\ x+y < 1 \\ 0 \quad , & \text{elsewhere} \end{cases}$$

Find $E(X)$ and $E(Y)$.

9. Answer any *one* of the following questions : 8

(a) (i) A fair die is tossed. Let X denotes twice the number appearing and Y denotes 1 or 3, depending on whether an odd or an even number appears. Find the distribution, expectation and variance of $(X+Y)$. 5

(ii) The probability density function of a two-dimensional random variable (X, Y) is given by

$$f(x, y) = \begin{cases} x+y, & 0 < x+y < 1 \\ 0 \quad , & \text{elsewhere} \end{cases}$$

Evaluate $P\left(X < \frac{1}{2}, Y > \frac{1}{4}\right)$. 3

(b) Two random variables X and Y have the following joint probability density function :

$$f(x, y) = \begin{cases} 2-x-y, & 0 \le x \le 1,\ 0 \le y \le 1 \\ 0 \quad , & \text{otherwise} \end{cases}$$

Find—

(i) marginal probability density function of X and Y;

(ii) conditional density functions;

(iii) var(X) and var(Y).

UNIT—IV

10. Answer any *four* of the following questions : 1×4=4

(a) Define covariance of a bivariate data.

(b) Give a interpretation of correlation coefficient.

(c) Find the coefficient of correlation, when

$\text{cov}(X, Y) = -16 \cdot 5,\ \text{var}(X) = 2 \cdot 89,\ \text{var}(Y) = 100$

(d) Write one property of regression coefficient.

(e) Is the following statement correct? Give reasons :

The regression coefficient of x on y is 3·2 and that of y on x is 8.

11. Answer any *one* of the following questions : 2

(a) Find the equation of the regression line of Y on X from the observations (1, 4), (2, 8), (3, 2) and (4, 12).

(b) Define bivariate normal distribution function.

12. Answer any *one* of the following questions : 8

(a) *(i)* Equations of two regression lines are $4x + 3y + 7 = 0$ and $3x + 4y + 8 = 0$. Find—

(1) mean of x and mean of y;

(2) regression coefficients b_{yx} and b_{xy};

(3) correlation coefficient between x and y. 5

(ii) Show that the correlation coefficient is independent of change of origin and scale. 3

(b) Derive the expression for moment generating function of a bivariate normal distribution.

UNIT—V

13. Answer any *four* of the following as directed : 1×4=4

(a) State central limit theorem.

(b) Let $X_1, X_2, \cdots, X_n$ be a random sample from the population of X with $E(X) = \mu$ and $\text{var}(X) = \sigma^2$. Also let $\varepsilon > 0$ and $\delta (0 < \delta < 1)$ be given number, then

$$P(|\overline{X_n} - \mu| < \varepsilon) \geq 1 - \delta$$

for all $n > \dfrac{\sigma^2}{\varepsilon^2 \delta}$, where $\overline{X_n}$ = ______.

(Fill in the blank)

(c) State Markoff's theorem.

(d) Define Bernoulli's law of large numbers.

(e) State Khintchine's theorem.

14. Answer any *one* of the following questions : 2

(a) For geometric distribution

$$p(x) = 2^{-x}; \; x = 1, 2, \cdots$$

and $E(X) = 2$, prove that Chebyshev's inequality gives

$$P(|X - 2| \leq 2) > \frac{1}{2}$$

Also find $\text{var}(X)$.

(b) Explain when a sequence of independent and identically distributed random variables is consistent.

15. Answer any *one* of the following questions : 8

(a) *(i)* If $X_1, X_2, \cdots, X_n$ are identically distributed random variables such that for each X_k,

$$P(X_k = x) = \frac{c^x e^{-c}}{\lfloor x}, \; x = 0, 1, 2, \cdots$$

and $c > 0$, find

$$P(X_1 + X_2 + \cdots + X_n)$$

and also calculate

$$\text{var}(X_1 + X_2 + \cdots + X_n)$$ 5

(ii) Let $X_1, X_2, \cdots, X_n$ be n random variables which are independently and identically distributed with mean μ and variance σ^2. Then find $E(\overline{X})$ and $\text{var}(\overline{X})$, where

$$\overline{X} = \frac{X_1 + X_2 + \cdots + X_n}{n}$$ 3

(b) Prove the central limit theorem.

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