TDC (CBCS) Odd Semester Exam., 2023

MATHEMATICS

(5th Semester)

Course No. : MTMDSE-502T

Full Marks : 70

Pass Marks : 28

Time : 3 hours

The figures in the margin indicate full marks for the questions

Candidates have to answer *either* from Option—A *or* Option—B

OPTION—A

Course No. : MTMDSE-502T (A)

(Analytical Geometry)

SECTION—A

Answer any *twenty* of the following questions : 1×20=20

Unit—I

1. When the equation $ax^2+2hxy+by^2=0$ represents a pair of imaginary straight lines?

2. What will be the two invariants when the expression $ax^2+2hxy+by^2$ changes to $a'x'^2+2h'x'y'+b'y'^2$ by an orthogonal transformation?

3. What is the angle between the pair of lines given by the equation $ax^2+2hxy+by^2=0$?

4. What is the condition that the equation

$$ax^2+2hxy+by^2+2gx+2fy+c=0$$

represents two perpendicular straight lines?

5. Write down the equations to the bisectors of the angles between the lines represented by the equation $x^2+2y^2+4xy=0$.

Unit—II

6. Define orthogonal circles.

7. Define coaxial circles.

8. How many tangents can be drawn from a given point to a parabola?

9. What is the condition of tangency of a line $y=mx+c$ to a circle $x^2+y^2=a^2$?

10. Find the equation of the tangent at the point (0, 4) to the ellipse $16x^2+25y^2=400$.

Unit—III

11. Define pole and polar in a circle.

12. Is the statement "If the polar of P with respect to conic passes through Q, then the polar of Q also passes through P" true?

13. Write the polar equation of a conic.

14. Find the polar equation of a parabola whose latus rectum is 16.

15. Discuss the nature of the conic

$$\frac{15}{r}=3-4\cos\theta$$

Unit—IV

16. What is the shortest distance between two coplanar lines?

17. Write the equation of a plane in intercept form.

18. Write the centre and radius of the sphere

$$x^2 + y^2 + z^2 - ax - by - cz = 0$$

19. Define great circle.

20. Find the equation of the tangent plane to the sphere whose centre is at the origin, radius 1 unit at (1, 0, 0).

Unit—V

21. Define cone.

22. Write down the equation of the cone whose vertex is the origin and the direction cosines of its generators satisfy the relation

$$5l^2 - 4m^2 + 9n^2 = 0$$

23. What do you mean by slant height of a cone?

24. What is generator of a cylinder?

25. Define right circular cylinder.

SECTION—B

Answer any *five* of the following questions : 2×5=10

Unit—I

26. Find the equation of the line $y = \sqrt{3}x$ when the axes are rotated through an angle $\pi / 3$.

27. If the equation

$$ax^2 + 3xy - 2y^2 - 5x + 5y + c = 0$$

represents two straight lines perpendicular to each other, then find a and c.

Unit—II

28. Find the radical axis of the circles

$$x^2 + y^2 + 2x + 4y + 7 = 0$$

and $$2x^2 + 2y^2 + 2x + 4y + 7 = 0.$$

29. Prove that the line $lx + my + n = 0$ touches the hyperbola

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$$

if $n^2 = a^2l^2 - b^2m^2$.

Unit—III

30. Find the equation of the polar of the point (2, 3) with respect to the circle

$$x^2 + y^2 - 2x - 4y + 1 = 0$$

31. Find the point on the curve

$$\frac{14}{r} = 3 - 8\cos\theta$$

whose radius vector is 2.

Unit—IV

32. Find the shortest distance between y-axis and the line

$$\frac{x - x_1}{l} = \frac{y - y_1}{m} = \frac{z - z_1}{n}$$

33. Find the equation of the tangent plane(s) to the sphere $x^2 + y^2 + z^2 = 36$ parallel to the plane $x - 2y + 3z = 0$.

Unit—V

34. Find the equation of the cone whose vertex is the origin and base is the circle $x = a$, $y^2 + z^2 = b^2$.

35. Find the equation of the right circular cylinder whose axis is

$$\frac{x}{1} = \frac{y}{-2} = \frac{z}{2}$$

and radius 2.

SECTION—C

Answer any *five* of the following questions : 8×5=40

Unit—I

36. *(a)* If $ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$ represents two straight lines equidistant from origin, show that

$$f^4 - g^4 = c(bf^2 - ag^2)$$ 4

(b) Prove that the pair of straight lines joining the origin to the other two points of intersection of the curve $ax^2 + 2hxy + by^2 + 2gx = 0$ and

$$a'x^2 + 2h'xy + b'y^2 + 2g'x = 0$$

will be at right angles if

$$g(a' + b') = g'(a + b)$$ 4

37. *(a)* Find the new origin (α, β) without changing the direction of the axes such that the equation

$$5x^2 - 2y^2 - 30x + 8y = 0$$

is transformed to the form

$$ax'^2 + by'^2 = 1$$ 4

(b) Prove that the two pairs of lines

$$ax^2 + 2hxy + by^2 = 0$$

and $(a-b)(x^2 - y^2) + 4hxy = 0$ have the same bisectors. 4

Unit—II

38. *(a)* If $S = 0$, $S' = 0$ be two circles of radii r and R, then prove that the circles

$$\frac{S}{r} \pm \frac{S'}{R} = 0$$

will cut orthogonally. 4

(b) Show that the product of the length of the perpendicular drawn from the foci on any tangent to the ellipse

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

is b^2. 4

39. *(a)* Find the point of the parabola $y^2 = 8x$ at which the normal is inclined at 60° to the axis of the parabola. 4

(b) Find the equation of pair of tangents to the circle $x^2 + y^2 = 9$ drawn from the point (10, 0). 4

Unit—III

40. *(a)* Find the equation of polar of the point (x_1, y_1) with respect to the circle

$$x^2 + y^2 + 2gx + 2fy + c = 0$$ 4

(b) If PSP' and QSQ' be two perpendicular focal chords of a conic

$$\frac{l}{r} = 1 + e\cos\theta$$

then show that

$$\frac{1}{PP'} + \frac{1}{QQ'} = \text{constant}$$ 4

41. *(a)* A conic is described having the same focus and eccentricity as the conic

$$\frac{l}{r} = 1 + e\cos\theta$$

and the two conics touch at $\theta = \alpha$. Prove that the length of its latus rectum is

$$\frac{2l(1-e^2)}{e^2 + 2e\cos\alpha + 1}$$ 4

(b) Find the condition that the line

$$\frac{l}{r} = A\cos\theta + B\sin\theta$$

may be a tangent to the conic

$$\frac{l}{r} = 1 + e\cos\theta$$ 4

Unit—IV

42. *(a)* Find the length and equations of the shortest distance between

$$3x - 9y + 5z = 0 = x + y - z$$

and $6x + 8y + 3z - 13 = 0 = x + 2y + z - 3$. 5

(b) A plane passes through a fixed point (a, b, c) and cuts the axes in A, B, C. Show that the locus of the centre of the sphere $OABC$ is

$$\frac{a}{x} + \frac{b}{y} + \frac{c}{z} = 2$$ 3

43. *(a)* Show that the lines

$$\frac{x-1}{1} = \frac{y-1}{2} = \frac{z-1}{3}$$

and $\frac{x-4}{2} = \frac{y-6}{3} = \frac{z-8}{4}$ are coplanar. Also find their point of intersection, if any. 4

(b) Prove that the plane $2x - 2y + z + 12 = 0$ touches the sphere

$$x^2 + y^2 + z^2 - 2x - 4y + 2z - 3 = 0$$

and find the point of contact. 4

Unit—V

44. *(a)* Find the equation of the cone with vertex at (2, 1, 0), the guiding curve being $2x^2 + 3y^2 - 1 = 0 = z$. 4

(b) What will be the equation of the right circular cylinder having radius 3 units and axis passing through (2, 1, 0) and having direction ratios −1, 2, 3? 4

45. *(a)* A variable plane is parallel to the given plane

$$\frac{x}{a}+\frac{y}{b}+\frac{z}{c}=0$$

and meets the axes in A, B and C respectively. Prove that the circle ABC lies on the cone

$$yz\left(\frac{b}{c}+\frac{c}{b}\right)+zx\left(\frac{a}{c}+\frac{c}{a}\right)+xy\left(\frac{a}{b}+\frac{b}{a}\right)=0$$ 4

(b) Find the equation of the cylinder generated by the lines parallel to the line

$$\frac{x}{1}=\frac{y}{-2}=\frac{z}{5}$$

the guiding curve being the conic $y^2=8z,\ x=0$. 4

OPTION—B

Course No. : MTMDSE–502T (B)

(Probability and Statistics)

SECTION—A

Answer any *twenty* of the following questions : 1×20=20

Unit—I

1. If A and B are any two independent events and

$$P(A)=\frac{1}{2},\ P(B)=\frac{1}{3}$$

find $P(A\cap B)$.

2. What is meant by random variable?

3. Let A and B be the possible outcomes of an experiment and suppose $P(A)=0\cdot4$, $P(A\cup B)=0\cdot7$ and $P(B)=p$. For what choice of p are A and B mutually exclusive?

4. Define probability mass function.

5. State the additive law of probability.

Unit—II

6. Comment on the following :

"The mean of a binomial distribution is 3 and variance is 4."

7. Write a difference between binomial distribution and Poisson distribution.

8. If

$$X \sim B\left(5, \frac{1}{2}\right)$$

find mean and variance.

9. Write one characteristic of the normal distribution.

10. Give an example of a discrete distribution.

Unit—III

11. The joint probability density function of a two-dimensional random variable (X, Y) is given by

$$f(x, y) = 2; \quad 0 < x < 1, \quad 0 < y < x$$
$$= 0; \quad \text{elsewhere}$$

Find the marginal density function of X.

12. Find the probability, $P(X+Y=8)$, if two fair dice are tossed simultaneously and

X : number on the first die

Y : number on the second die

13. State the conditions of independence of two random variables X and Y with joint p.d.f. $f_{XY}(x, y)$.

14. If $P_1(x)$ and $P_2(y)$ be the marginal probability function of two independent discrete random variables X and Y, then the joint probability function is _____.

(Fill in the blank)

15. Comment on the following statement writing True or False :

If X and Y are independent random variables and $h(X)$ is a function of X alone and $k(Y)$ is a function of Y alone, then

$$E[h(X)k(Y)] = E[h(X)]E[k(Y)]$$

Unit—IV

16. Define bivariate normal distribution.

17. If $(X, Y) \sim BVN(0, 0, 1, 1, \rho)$, find the regression line of Y on X.

18. If $(X, Y) \sim BVN(\mu_1, \mu_2, \sigma_1^2, \sigma_2^2, \rho)$, write the condition of independence of X and Y.

19. If (X, Y) possesses a bivariate normal distribution, define marginal distribution of X.

20. If $f_1(x, y)$ is the p.d.f. of $BVN(0, 0, 1, 1, \rho)$ and $f_2(x, y)$ is the p.d.f. of $BVN(0, 0, 1, 1, -\rho)$, then write the formula for joint distribution of X and Y.

Unit—V

21. Write the condition under which the law of large numbers holds.

22. What is meant by convergence in probability?

23. If $X_1, X_2, \cdots, X_n$ are independent and identically distributed $B(r, p)$, then what is $E(X_1 + X_2 + \cdots + X_n)$?

24. If X is the number of success in n Bernoulli trials with constant probability p of success for each trial, then

$$\underset{n\to\infty}{\text{Lt}}\ P\left\{\left|\frac{x}{n} - p\right| < \varepsilon\right\}$$

is ______.

(Fill in the blank)

25. Under what condition both the central limit theorem and weak law of large numbers hold for a sequence of random variables $\{X_n\}$?

SECTION—B

Answer any *five* of the following questions : 2×5=10

Unit—I

26. If A and B are two events with $P(A) = 0 \cdot 4$, $P(A \cup B) = 0 \cdot 7$ and $P(B) = 0 \cdot 6$, find the value of $P(A \mid B)$.

27. If the distribution function of X is $F(x) = 1 - e^{-x}$, $0 \le x < \infty$, find the p.d.f. of X.

Unit—II

28. Suppose that X has a Poisson distribution. If $P(X = 1) = P(X = 2)$, find the mean of the distribution.

29. If X has a uniform distribution in (a, b), then find the mean.

Unit—III

30. Two fair dice are tossed simultaneously. Let

X : number on the first die

Y : number on the second die

Write down the sample space.

31. Let X and Y be jointly distributed with p.d.f.

$$f(x, y) = \begin{cases} \frac{1}{4}(1 + xy), & |x| < 1, \quad |y| < 1 \\ 0 \quad , & \text{otherwise} \end{cases}$$

Examine if X and Y are independent.

Unit—IV

32. For two-dimensional random variables (X, Y), if $\sigma^2_{X=9}$ and regression equations

$$8X - 10Y + 66 = 0$$
$$40X - 18Y = 214$$

find the mean values of X and Y.

33. Prove or disprove that correlation coefficient is the geometric mean between the regression coefficients.

Unit—V

34. State the weak law of large numbers.

35. For a random variable X, $E(X) = 5$, $V(X) = 3$, what is the least value of probability $\{|x - 5| < 3\}$?

SECTION—C

Answer any *five* of the following questions : 8×5=40

Unit—I

36. *(a)* A can hit a target 4 times in 5 shots; B 3 times in 4 shots; C twice in 3 shots. They fire a volley. What is the probability that at least two shots it? 5

(b) An urn contains 10 white and 3 black balls while another urn contains 3 white and 5 black balls. Two balls are drawn from the first urn and put into the second urn and then a ball is drawn from the latter. What is the probability that it is a while ball? 3

37. *(a)* If X is a random variable with p.m.f. $P(X = x) = q^x p$, $x = 0, 1, 2, \cdots, \infty$, $q = 1 - p$, find the moment generating function of X and hence find mean of X. 5

(b) If a random variable X has the density function

$$f(x) = \begin{cases} \frac{1}{4}, & -2 < x < 2 \\ 0, & \text{elsewhere} \end{cases}$$

obtain $E(x)$, $P(|X| > 1)$. 3

Unit—II

38. *(a)* Prove that limiting case of binomial distribution is Poisson distribution. 6

(b) If X has a uniform distribution in (a, b), find the moment generating function. 2

39. *(a)* In a normal distribution, 31% of the items are under 45 and 8% are over 64. Find the mean and standard deviation of the distribution. 5

(b) What is meant by a geometric distribution? Find the mean of the distribution. 3

Unit—III

40. The joint probability distribution of a pair of random variables is given by the following table :

Y \ X	1	2	3
1	0·1	0·1	0·2
2	0·2	0·3	0·1

Find—

(a) the marginal distributions;

(b) the conditional distribution of X given $Y = 1$;

(c) the conditional distribution of Y given $X = 2$;

(d) $P\{(X + Y) < 4\}$. 8

41. If

$$f(x, y) = e^{-(x+y)}; \quad x \geq 0, \ y \geq 0$$
$$= 0 \quad ; \quad \text{elsewhere}$$

is the joint probability density function of random variables X and Y, then

(a) find the marginal density functions of X and Y;

(b) find $P(X < 1)$, $P(X > 1)$;

(c) find the conditional density function of Y given $X = x$;

(d) examine the independence of X and Y. 8

Unit—IV

42. If the joint p.d.f. of random variables X and Y is given by

$$f(x, y) = \frac{1}{3}(x + y); \quad 0 < x < 1, \ 0 < y < 2$$
$$= 0 \quad ; \quad \text{otherwise}$$

determine—

(a) the correlation coefficient between X and Y; 5

(b) the two lines of regression. 3

43. The bivariate random variable (X, Y) has a bivariate normal distribution with means 5 and 10; standard deviations 1 and 5 with the correlation coefficient ρ.

(a) If $\rho > 0$, find ρ when

$$P(4 < Y < 16 \mid X = 5) = 0 \cdot 954$$ 4

(b) If $\rho = 0$, find $P(X + Y \leq 16)$. 4

Unit—V

44. *(a)* State and prove the Chebyshev's inequality. 5

(b) If X is a random variable subject to

$$E(X) = 3 \text{ and } E(X^2) = 13$$

use Chebyshev's inequality to determine the lower bound for the probability $P\{-2 \leq X \leq 8\}$. 3

45. Let $X_1, X_2, \cdots$ be independent and identically distributed Poisson variates with parameter λ. Then obtain—

(a) $E(X_1 + X_2 + \cdots + X_n)$ and $V(X_1 + X_2 + \cdots + X_n)$ 4

(b) $P(120 \leq X_1 + X_2 + \cdots + X_n \leq 160)$ $(\lambda = 2,\ n = 75)$ 4

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