# 2019/TDC/ODD/SEM/MTMDSC/ MTMGE–301T/260

## TDC (CBCS) Odd Semester Exam., 2019

## MATHEMATICS

**( 3rd Semester )**

Course No. : MTMDSC/MTMGE–301T

**( Real Analysis )**

*Full Marks* : 70

*Pass Marks* : 28

*Time* : 3 hours

*The figures in the margin indicate full marks for the questions*

### UNIT—I

**1.** Answer any *four* of the following : $1\times4=4$

*(a)* Define finite set and give an example.

*(b)* Find a lower bound of the set of positive real numbers.

*(c)* Give an example of a countable collection of finite sets whose union is not finite.

(d) Let

$$S = \left\{1 - \frac{(-1)^n}{n};\ n \in \mathbb{N}\right\}$$

Find sup $S$.

(e) Give an example of a set which is bounded below but not bounded above.

2. (a) Show that the greatest lower bound of a set bounded below is unique. 2

Or

(b) Show that the set of all odd natural numbers is countable.

3. (a) Prove that a countable union of countable sets is countable. 4

(b) State and prove Archimedean property of $\mathbb{R}$. 1+3=4

**OR**

4. (a) Prove that the set of rational numbers is not order complete. 5

(b) Show that the supremum of a nonempty set $S$ of real numbers, whenever it exists, is unique. 3





UNIT—II

5. Answer any *four* of the following : 1×4=4

(a) What is the derived set of $Q$?

(b) Define limit point of a subset of $\mathbb{R}$.

(c) Give an example of a set which is neither closed nor open in $\mathbb{R}$.

(d) Obtain the derived set of the set

$$\left\{\frac{1}{n} : n \in \mathbb{N}\right\}$$

(e) Give an example of an open set which is not an interval.

6. (a) Obtain the derived set of the following sets : 2

(i) $\{1, 2, 3, 4, 5, 6\}$

(ii) $\{1, 2, 3, 4, \cdots, 500\}$

Or

(b) Prove that the union of two open intervals is not necessarily an open interval.

7. (a) Prove that the intersection of any finite number of open sets is open. 4

(b) State and prove Bolzano-Weierstrass theorem. (for sets) 4

**OR**

8. (a) Prove that a set is closed if its complement is open. 3

(b) If a sequence of closed intervals $[a_n, b_n]$ is such that each member $[a_{n+1}, b_{n+1}]$ is contained in the preceding one $[a_n, b_n]$ and $\lim(b_n - a_n) = 0$, then prove that there is one and only one point common to all the intervals of the sequence. 5

UNIT—III

9. Answer any *four* of the following : 1×4=4

(a) Show that the sequence $\{a_n\}$, where $a_n = \frac{n+1}{n}$ is convergent.

(b) Show that the sequence $\{x_n\}$, where $x_n = n^2$ is monotonically increasing.

(c) Define bounded sequence with example.

(d) Give an example of two divergent sequences $X$ and $Y$ such that their product $XY$ converge.

(e) Give an example of a bounded sequence that is not a Cauchy sequence.





10. (a) Show that every bounded sequence may not be a convergent sequence. 2

*Or*

(b) Applying Squeeze theorem, show that $\underset{n\to\infty}{\text{Lt}} \frac{\sin n}{n} = 0$

11. (a) Prove that every convergent sequence is bounded. 4

(b) Show that the sequence $\{S_n\}$, where $S_n = 1 + \frac{1}{2} + \frac{1}{3} + \cdots + \frac{1}{n}$ cannot converge. 4

**OR**

12. (a) State and prove Squeeze theorem. 4

(b) Define monotone sequence. Show that the sequence

$$S_n = \frac{1}{1!} + \frac{1}{2!} + \cdots + \frac{1}{n!}, \ \forall\, n \in \mathbb{N}$$

is convergent. 1+3=4

UNIT—IV

13. Answer any *four* of the following : 1×4=4

(a) Give an example of a convergent series which is not absolutely convergent.

(b) Justify if the series $\Sigma \frac{1}{3^n}$ is convergent or divergent.

(c) Show that the series

$$\frac{1}{2}+\frac{2}{3}+\frac{3}{4}+\cdots$$

is not convergent.

(d) Can you give an example of a convergent series $\Sigma x_n$ and a divergent series $\Sigma y_n$ such that $\Sigma(x_n + y_n)$ is convergent?

(e) Give an example of a conditional convergent series.

**14.** (a) Test the convergence of $\Sigma x_n$, where $x_n = \frac{n}{n^2+1}$. 2

*Or*

(b) Prove or disprove the series $\Sigma u_n$ is convergent if $\underset{n\to\infty}{\text{Lt}}\ u_n = 0$

**15.** (a) Test the convergence of the following series : 2+3=5

(i) $\sum(\sqrt{n^4+1}-\sqrt{n^4-1})$

(ii) $\frac{x}{1\cdot 2}+\frac{x^2}{2\cdot 3}+\frac{x^3}{3\cdot 4}+\cdots$

(b) Show that the series

$$x+\frac{x^2}{2!}+\frac{x^3}{3!}+\cdots$$

converges absolutely for all values of $x$. 3





OR

**16.** (a) If $\Sigma u_n$ is a positive term series such that

$$\underset{n\to\infty}{\text{Lt}}\ \frac{u_n+1}{u_n}=l$$

then the series

(i) converges, if $l<1$;

(ii) diverges, if $l>1$. 5

(b) Test the convergence of the series

$$1+\frac{x^2}{2!}+\frac{x^4}{4!}+\frac{x^6}{6!}+\cdots$$ 3

## UNIT—V

**17.** Answer any *four* of the following : 1×4=4

(a) Write the sequential criterion for limit of a function.

(b) Using sequential criterion, show that $\underset{x\to 0}{\text{Lt}}\ \frac{1}{x}$ does not exist.

(c) Give an example of a function which is not continuous at any point of $\mathbb{R}$.

(d) Give an example of a function $f:[0, 1]\to\mathbb{R}$ that is discontinuous at every point of [0, 1] but such that $|f|$ is continuous on [0, 1].

(e) Define bounded function with example.

**18.** (a) For what value of $a$ the function

$$f(x) = \begin{cases} x+3, & x \geq 1 \\ ax^2+8, & x < 1 \end{cases}$$

is continuous on $\mathbb{R}$? 2

*Or*

(b) Show that

$$\underset{x \to 0}{\text{Lt}} \sin \frac{1}{x}$$

does not exist.

**19.** (a) Prove that a function $f$ defined on an interval $I$ is continuous at a point $c \in I$ if and only if for every sequence $\{c_n\}$ in $I$ converging to $c$, we have $\underset{n \to \infty}{\text{Lt}} f(c_n) = f(c)$. 5

(b) Show that the function $f$ defined on $\mathbb{R}$ by

$$f(x) = \begin{cases} 1, & \text{when } x \text{ is rational} \\ 0, & \text{when } x \text{ is irrational} \end{cases}$$

is not continuous at any point of $\mathbb{R}$. 3

**OR**

**20.** (a) If $f$, $g$ be two functions continuous at a point $c$, then show that the functions $f+g$, $f_g$ are also continuous at $c$. 2+2=4

(b) Prove that if a function is continuous in a closed and bounded interval, then it is bounded therein. 4

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