



**2023/TDC(CBCS)/EVEN/SEM/
MTMDSC/GE-201T/030**

TDC (CBCS) Even Semester Exam., 2023

MATHEMATICS

(2nd Semester)

Course No. : MTMDSC/GE-201T

(Differential Equations)

Full Marks : 70

Pass Marks : 28

Time : 3 hours

*The figures in the margin indicate full marks
for the questions*

SECTION—A

Answer any *twenty* questions :

1×20=20

1. Is the differential equation

$$(ax + hy + g)dx + (hx + by + f)dy = 0$$

exact?

2. Find the value of $d\left(\frac{y^2}{x}\right)$.

3. Define integrating factor.



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4. If

$$\frac{1}{M} \left(\frac{\partial N}{\partial x} - \frac{\partial M}{\partial y} \right) = f(y)$$

what is the integrating factor of $Mdx + Ndy = 0$?

5. Write the equation

$$\left(\frac{dy}{dx} \right)^2 (x^2 - a^2) - 2 \left(\frac{dy}{dx} \right) xy + y^2 - b^2 = 0$$

in Clairaut's form.

6. Give an example of 2nd-order linear differential equation with constant coefficient.

7. Define linearly independent solution.

8. Under what condition, $y = e^x$ is the solution of

$$\frac{d^2y}{dx^2} + P \frac{dy}{dx} + Qy = 0 ?$$

9. Verify that e^{2x} is one of the solutions of

$$\frac{d^3y}{dx^3} - 8y = 0$$

10. Find the value of $w(x, 2x^2)$.

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11. Solve :

$$\frac{d^2y}{dx^2} - 4y = 0$$

12. Solve :

$$\frac{d^2y}{dx^2} = e^{ax}$$

13. Find the CF of the differential equation

$$(D^2 + 6D + 5)y = 16e^{3x}, \text{ where } D = \frac{d}{dx}.$$

14. Find the PI of the differential equation

$$(D^2 - 1)y = \sin 2x, \text{ where } D = \frac{d}{dx}.$$

15. If $f(D^2) = \cos ax$, write PI, provided $f(-a^2) \neq 0$.

16. Define total differential equation.

17. Is the equation $ydx + xdy + zdz = 0$ integrable?

18. Solve :

$$\frac{dx}{z} = \frac{dy}{0} = \frac{dz}{-x}$$

19. Give an example of a Cauchy-Euler equation.

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20. Define ordinary simultaneous differential equations.
21. Define degree of a partial differential equation.
22. Write the degree of the partial differential equation

$$\left(\frac{\partial z}{\partial x}\right)^2 + \frac{\partial^3 z}{\partial y^3} = 2x \frac{\partial z}{\partial x}$$

23. Write the order of the partial differential equation

$$\frac{\partial^2 z}{\partial x^2} = \left(1 + \frac{\partial z}{\partial y}\right)^{\frac{1}{2}}$$

24. Define linear partial differential equation.
25. Whether the partial differential equation $z\left(\frac{\partial z}{\partial x}\right) + \frac{\partial z}{\partial y} = x$ is linear or non-linear?

SECTION—B

Answer any five questions : 2×5=10

26. Solve : $(e^y + 1)\cos x dx + e^y \sin x dy = 0$

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(5)

27. Find integrating factor of the differential equation

$$(x^2 + y^2 + 1)dx - 2xydy = 0$$

28. Show that $\sin x$ and $\sin x - \cos x$ are linearly independent solution of

$$\frac{d^2 y}{dx^2} + y = 0$$

29. Show that the Wronskian of $e^{ax} \cos bx$ and $e^{ax} \sin bx$, ($b \neq 0$) is be^{2ax} .

30. Solve $(D^2 + 4)y = x^2$, where $D = \frac{d}{dx}$.

31. Solve $(D^2 - 4D + 4)y = x^3 e^{2x}$, where $D = \frac{d}{dx}$.

32. Show that

$$(2x + y^2 + 2xz)dx + 2xydy + x^2 dz = 0$$

is integrable.

33. Solve :

$$\frac{x dx}{y^2 z} = \frac{dy}{xz} = \frac{dz}{y^2}$$

34. Eliminate the arbitrary constants a and b from $z = (x - a)^2 + (y - b)^2$ to form the partial differential equation.

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(Turn Over)



(6)

35. Form a partial differential equation by eliminating arbitrary constants h and k from $z = hx + ky + hk$.

SECTION—C

Answer any five questions : 8×5=40

36. (a) State and prove the necessary and sufficient condition for the differential equation $Mdx + Ndy = 0$ to be exact. 5

(b) Solve $P(P^2 + xy) = P^2(x + y)$, where $P = \frac{dy}{dx}$. 3

37. (a) Solve $(x^3 - 2y^2)dx + 2xydy = 0$. 3

(b) Solve $y = px + \sqrt{a^2 p^2 + b^2}$, where $p = \frac{dy}{dx}$. 5

38. (a) Determine the differential equation with the linearly independent solution x , x^2 and x^3 . 3

(b) Show that the Wronskian of the functions x^2 , $x^2 \log x$ is non-zero. Can these functions be independent solutions of an ordinary differential equation? If so, determine this equation. 5

(7)

39. (a) Verify that the functions $\sin x$, $\cos x$ and $\sin 2x$ are linearly independent. 4

(b) Prove that the Wronskian of the functions $e^{m_1 x}$, $e^{m_2 x}$, $e^{m_3 x}$ is equal to $(m_1 - m_2)(m_2 - m_3)(m_3 - m_1)e^{(m_1 + m_2 + m_3)x}$. 4

40. (a) Solve $(x^2 D^2 + 3xD + 1)y = \frac{1}{(1-x)^2}$, where $D = \frac{d}{dx}$. 4

(b) Solve $(D^2 + a^2)^2 y = \sin ax$, where $D = \frac{d}{dx}$. 4

41. (a) Solve $(D^2 - 6D + 9)y = 6e^{3x}$, where $D = \frac{d}{dx}$. 4

(b) Solve : $\frac{d^2 y}{dx^2} - 2 \frac{dy}{dx} + 4y = e^x \cos x$ 4

42. (a) Solve : $x^3 \frac{d^3 y}{dx^3} + x^2 \frac{d^2 y}{dx^2} = x$ 4



(b) Solve :

$$\frac{dx}{x^2 + 2y^2} = \frac{dy}{-xy} = \frac{dz}{xz}$$

4

43. (a) Prove that the necessary condition for integrability of total differential equation $Pdx + Qdy + Rdz = 0$ is

$$P\left(\frac{\partial Q}{\partial z} - \frac{\partial R}{\partial y}\right) + Q\left(\frac{\partial R}{\partial x} - \frac{\partial P}{\partial z}\right) + R\left(\frac{\partial P}{\partial y} - \frac{\partial Q}{\partial x}\right) = 0$$

5

(b) Solve :

$$\frac{dx}{dt} = 2y + x, \quad \frac{dy}{dt} = y$$

3

44. (a) Find the partial differential equation of the set of all right circular cones whose axes coincide with z-axis.

4

(b) Form a partial differential equation by eliminating the arbitrary function ϕ

$$\text{from } \phi(x + y + z, x^2 + y^2 - z^2) = 0.$$

4

45. (a) Form a partial differential equation by eliminating arbitrary functions f and F from $y = f(x - at) + F(x + at)$.

4

(b) Find the partial differential equation of all planes which are at a constant distance a from the origin.

4

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