2019/TDC/EVEN/MTMDSC/
MTMGEC–201T/031

TDC (CBCS) Even Semester Exam., 2019

MATHEMATICS

(2nd Semester)

Course No. : MTMDSC–201T/MTMGEC–201T

(Differential Equation)

Full Marks : 70
Pass Marks : 28

Time : 3 hours

The figures in the margin indicate full marks for the questions

UNIT—I

1. Answer the following questions : 1×2=2

(a) Define exact differential equation.

(b) Define Clairaut's equation.

Answer *either* (a) and (b) *or* (c) and (d) :

2. (a) Prove that the necessary condition for a differential equation $Mdx + Ndy = 0$ to be exact is that

$$\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$$ 3

(b) Solve the equation $y = xp + f(p)$, where $p = \frac{dy}{dx}$. 3

Or

(c) Show that the equation

$$(e^y + 1)\cos x\, dx + e^y \sin x\, dy = 0$$

is exact and hence solve it. 3

(d) Solve $p^2 + 2px + py + 2xy = 0$, where $p = \frac{dy}{dx}$. 3

Answer *either* (a) and (b) *or* (c) and (d) :

3. (a) Solve

$$(x^3 + y^3)dx - xy^2 dy = 0$$ 3

(b) Explain the method of solving the differential equation of the form

$$y = f(x, p),\ p = \frac{dy}{dx}$$ 3

Or

(c) Find the integrating factor of the equation $(x^2 + y^2 + x)dx - xy\, dy = 0$. 2

(d) Solve

$$y + px = p^2 x^4,\ p = \frac{dy}{dx}$$ 4

UNIT—II

4. Answer the following questions : 1×2=2

(a) Define Wronskian of n functions.

(b) State the basic theory of linear homogeneous differential equation.

5. (a) Define linearly dependent and independent set of functions. Prove that e^x, e^{-x} and e^{2x} are the linearly independent solution of

$$\frac{d^3y}{dx^3} - 2\frac{d^2y}{dx^2} - \frac{dy}{dx} + 2y = 0$$

Hence write its general solution. 2+4=6

Or

(b) Write a differential equation of second order in non-homogeneous form. Also solve the equation

$$\frac{d^2y}{dx^2} + 5\frac{dy}{dx} + 6y = e^x,\ y(0) = 5,\ y'(0) = 7$$

2+4=6

6. (a) Prove that the two solutions of the linear differential equation $y''(x) + Py'(x) + Qy = 0$ are linearly dependent if and only if their Wronskian vanishes identically, where P, Q are either constants or functions of x alone. 6

Or

(b) Given that

$$e^{-x}, e^{3x} \text{ and } e^{4x}$$

are all solutions of

$$\frac{d^3y}{dx^3} - 6\frac{d^2y}{dx^2} + 5\frac{dy}{dx} + 12y = 0$$

Show that they are linearly independent on the interval $-\infty < x < \infty$ and write the general solution. 6

UNIT—III

7. Answer the following questions : 1×2=2

(a) Find complementary function of the given equation :

$$(D^2 + a^2)y = \cos ax$$

(b) Find particular integral of the given differential equations :

$$(D^2 + D + 1)y = e^x$$

8. *(a)* Solve the following equations : 3×2=6

(i) $\frac{d^2y}{dx^2} + 4y = e^x + \sin 2x$

(ii) $\frac{d^2y}{dx^2} + \frac{dy}{dx} - 2y = 0,$

when $x = 0, y = 3$ and $\frac{dy}{dx} = 0$

Or

(b) Solve

$$\frac{d^2y}{dx^2} + n^2y = \sec nx$$

using the method of variation of parameters. 6

9. *(a)* Discuss the method of solving a second order differential equation by variation of parameters. 6

Or

(b) Solve the following equations : 3×2=6

(i) $(D^2 - 2D + 5)y = 10\sin x$

(ii) $(D - 2)^2 y = x^2 e^{2x}$

UNIT—IV

10. Answer the following questions : 2×1=2

(a) Define total differential equation.

(b) How many arbitrary constants will have if a linear differential equation is of n-th order?

11. *(a)* Write down Cauchy-Euler equation. Also solve the differential equation $(x^2D^2 - xD + 2)y = x\log x$. 2+4=6

Or

(b) Write down the condition for the integrability of the equation $Pdx + Qdy + Rdz = 0$. Hence solve the differential equation

$$(yz + 2x)dx + (zx - 2z)dy + (xy - 2y)dz = 0$$

2+4=6

12. *(a)* Solve the following equation : 3×2=6

(i) $\frac{dx}{dt} = -wy$; $\frac{dy}{dt} = wx$

(ii) $\frac{dx}{x(y^2 - z^2)} = \frac{dy}{y(z^2 - x^2)} = \frac{dz}{z(x^2 - y^2)}$

Or

(b) Reduce the equation

$$(1 + x)^2 \frac{d^2y}{dx^2} + (1 + x)\frac{dy}{dx} + y = 4\cos\log(1 + x)$$

into Cauchy-Euler equation form and hence solve the equation. 2+4=6

UNIT—V

13. Answer the following questions : 1×2=2

(a) Find order and degree of the partial differential equation

$$\frac{\partial^2 z}{\partial x^2} = \sqrt{1 + \frac{\partial z}{\partial y}}$$

(b) Whether the following second-order partial differential equation is linear or not :

$$\frac{\partial^2 u}{\partial x^2} - 4\frac{\partial^2 u}{\partial x \partial y} + 4\frac{\partial^2 u}{\partial y^2} = 0$$

(Write yes or no)

14. *(a)* Form partial differential equations by eliminating arbitrary constants : 3+3=6

(i) $z = ax + a^2y^2 + b$

(ii) $z = (x - a)^2 + (y - b)^2$

Or

(b) Form partial differential equations by eliminating functions f and F : 3+3=6

(i) $y = f(x - at) + F(x + at)$

(ii) $z = f(x^2 - y^2)$

15. *(a)* Form a partial differential equation by eliminating function f from

$$z = e^{ax+by} f(ax - by)$$

Hence find its order and degree. 4+2=6

Or

(b) Form partial differential equation by eliminating A and p from

$$z = Ae^{pt} \sin px$$

Also find its degree and order. 4+2=6

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