2020 Pass Science

2020/TDC(CBCS)/ODD/SEM/
MTMDSC/GE–101T/326

TDC (CBCS) Odd Semester Exam., 2020
held in March, 2021

MATHEMATICS

(1st Semester)

Course No. : MTMDSC/MTMGE–101T

(Differential Calculus)

Full Marks : 70
Pass Marks : 28

Time : 3 hours

The figures in the margin indicate full marks for the questions

SECTION—A

Answer any *twenty* questions : 1×20=20

1. What is the difference between $x = a$ and $x \to a$?

2. $\lim_{x\to 2} \frac{x^2 - 4}{x - 2} = ?$

3. Write the value of

$$\underset{x \to a}{\text{Lt}} \frac{x^n - a^n}{x - a}$$

where n is rational and $a > 0$.

4. $\lim_{x \to \infty} \left(1 + \frac{1}{x}\right)^x = ?$

5. $\lim_{x \to 0} \frac{\log(1+x)}{x} = ?$

6. $\lim_{x \to 0} \frac{e^x - 1}{x} = ?$

7. Does $\lim_{x \to 0} \sin \frac{1}{x}$ exist?

8. $\lim_{x \to 0} \frac{(1+x)^n - 1}{x} = ?$

9. What is removable discontinuity of a function at a point?

10. What is discontinuity of first-kind of a function at a point?

11. What is discontinuity of second-kind of a function at a point?

12. Is the function $\cos \frac{1}{x}$ continuous at $x = 0$?

13. What is/are the point/points of discontinuity of the function $\frac{x^2 - 9}{x - 3}$?

14. What are the points of discontinuity of the greatest integer function $[x]$?

15. Define differentiability of a function at a point.

16. Is the function

$$f(x) = \begin{cases} x \sin \frac{1}{x}, & \text{if } x \neq 0 \\ 0, & \text{if } x = 0 \end{cases}$$

differentiable at $x = 0$?

17. What is the nth derivative of x^n?

18. What is the nth derivative of e^{ax}?

19. What is the nth derivative of $\sin 2x$?

20. Find $\frac{d^2y}{dx^2}$, if $y = \log \frac{1}{x}$.

21. State Leibnitz theorem on nth derivative of the product of two functions.

22. Find y_5, if $y = x^{10}$.

23. Find $\frac{\partial z}{\partial x}$, where $z = e^{\sin^2}(x^2 + y^2)$.

24. Define a homogeneous function of two variables.

25. What is the geometrical meaning of the derivative of a function at a point?

26. What is the equation of the normal to a curve at a given point?

27. What is the angle of intersection of two curves?

28. Write down the formula for Cartesian subtangent to a curve.

29. Define $\tan\phi$, where ϕ is the angle between the radius vector and the tangent.

30. Define curvature of a curve at a given point.

31. Write down the formula for radius of curvature of a curve $r = f(\theta)$ at any point θ.

32. Draw a sketch of the cardioid $r = a(1 + \cos\theta)$.

33. Interpret Rolle's theorem geometrically.

34. State Lagrange's mean value theorem.

35. What is Cauchy's form of remainder in Taylor's theorem?

36. State Maclaurin's infinite series.

37. Obtain the stationary points for the function $f(x) = 2x^3 - 21x^2 + 36x - 20$.

38. What is the minimum value of $\sin^2 x$?

39. What is the maximum value of $-x^2 + 4x + 5$?

40. Find $\lim_{x \to 1} \frac{\log x}{x-1}$.

SECTION—B

Answer any *five* questions : 2×5=10

41. State Cauchy's necessary and sufficient conditions for the existence of a limit.

42. Examine the existence of the limit

$$\lim_{x \to 0} \frac{e^{1/x} - e^{-1/x}}{e^{1/x} + e^{-1/x}}$$

43. Define the continuity of a function at a given point (ε-δ definition).

44. Show that the function $|x|$ is not differentiable at $x = 0$.

45. If $y = \sin^3 x$, then find y_n.

46. Find y_n, if $y = x^2 e^{ax}$.

47. Find the equation of the tangent to the curve

$$x = a(\theta + \sin\theta),\ y = b(1 - \cos\theta)$$

at any point θ.

48. Find the radius of curvature of the parabola $y^2 = 4ax$ at the vertex (0, 0).

49. Show that

$$\lim_{x \to 0} \frac{x - \sin x}{x^3} = \frac{1}{6}$$

50. Show that the function $x^3 - 6x^2 + 24x + 4$ has neither a maximum nor a minimum.

SECTION—C

Answer *any* **five** questions

51. *(a)* Using Cauchy's criterion, show that

$$\lim_{x \to 0} \cos\frac{1}{x}$$

does not exist. 3

(b) Evaluate :

$$\lim_{x \to 0} \frac{\operatorname{cosec} x - \cot x}{x}$$ 2

(c) Evaluate :

$$\lim_{x \to \infty} \frac{x^n}{x^n + 1}$$ 3

52. *(a)* Using ε-δ definition of limit, show that

$$\lim_{x \to 0} x \sin\frac{1}{x} = 0$$ 3

(b) Show that

$$\lim_{x \to 0} \frac{\sin x}{x} = 1$$

where x is in radian measure. 3

(c) Show that

$$\lim_{x \to 2} [x]$$

does not exist. 2

(Continued)

53. *(a)* Show that the function $f(x)$, defined by

$$f(x) = \begin{cases} -x, & \text{when } x \le 0 \\ x, & \text{when } 0 < x < 1 \\ 2 - x, & \text{when } x \ge 1 \end{cases}$$

is continuous at $x = 0$ and $x = 1$. 4

(b) If a function is differentiable at a point, then show that it is continuous there at. Also show by an example that the converse needs not be true. 3+1=4

54. *(a)* Show that the function

$$|x| + |x-1| + |x-2|$$

is continuous at $x = 0, 1, 2$. 4

(b) Show that the function $f(x)$, defined as

$$f(x) = \begin{cases} x^2 \cos\frac{1}{x}, & \text{when } x \ne 0 \\ 0, & \text{when } x = 0 \end{cases}$$

is differentiable at $x = 0$, but $f'(x)$ is not continuous there at. 4

55. *(a)* If $u = \sin ax + \cos ax$, then show that

$$u_n = a^n \sqrt{1 + (-1)^n \sin 2ax}$$ 4

(b) If $y = a\cos(\log x) + b\sin(\log x)$, then show that

$$x^2 y_{n+2} + (2n+1) x y_{n+1} + (n^2 + 1) y_n = 0$$ 4

(Turn Over)

56. *(a)* If $u = f(xyz)$, then show that
$$xu_x = yu_y = zu_z$$ 2

(b) If $u = \log(x^3 + y^3 + z^3 - 3xyz)$, then show that
$$\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} + \frac{\partial u}{\partial z} = \frac{3}{x+y+z}$$ 3

(c) If
$$u = \cos^{-1}\frac{x^2+y^2}{x+y}$$
then show that
$$x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} + \frac{1}{2}\cot u = 0$$ 3

57. *(a)* Prove that the condition that the line $lx + my = 1$ should touch the curve $(ax)^n + (by)^n = 1$ is
$$\left(\frac{l}{a}\right)^{\frac{n}{n-1}} + \left(\frac{m}{b}\right)^{\frac{n}{n-1}} = 1$$ 5

(b) Find the angle of intersection of the curves $x^2 - y^2 = 2a^2$ and $x^2 + y^2 = 4a^2$. 3

58. *(a)* Show that in the equiangular spiral $r = ae^{\theta \cot \alpha}$, the tangent is inclined at a constant angle to the radius vector. 3

(Continued)

(b) Find $\frac{ds}{d\theta}$ for the curve $r = ae^{\theta}$. 2

(c) Find the radius of curvature at any point (x, y) for the curve $x^{2/3} + y^{2/3} = a^{2/3}$. 3

59. *(a)* In Lagrange's mean value theorem
$$f(x+h) = f(x) + hf'(x+\theta h)$$
if $f(x) = Ax^2 + Bx + C$, $A \neq 0$, show that $\theta = \frac{1}{2}$. 4

(b) Show that $\sin x > x - \frac{x^3}{6}$, if $0 < x < \pi/2$. 4

60. *(a)* Find the maximum and minimum values of u, where
$$u = \frac{4}{x} + \frac{36}{y} \text{ and } x + y = 2$$ 4

(b) Find
$$\lim_{x \to 0} (\cot^2 x)^{\sin x}$$ 4

★ ★ ★