TDC (CBCS) Odd Semester Exam., 2021
held in March, 2022

MATHEMATICS

(5th Semester)

Course No. : MATSEC-501T

(Integral Calculus)

Full Marks : 50
Pass Marks : 20

Time : 3 hours

The figures in the margin indicate full marks for the questions

SECTION—A

Answer any *fifteen* from the following : 1×15=15

1. Write down the value of $\int \sec x\,dx$.

2. $\int \frac{dx}{a^2 - x^2} = ?$

3. What is the value of $\int x^n dx$ if $n = -1$?

4. Write down the value of $\int \sqrt{a^2 - x^2}\, dx$.

5. Write down the value of $\int_{-1}^{1} x^3\, dx$.

6. Show that

$$\int_a^b f(x)\, dx = -\int_b^a f(x)\, dx$$

7. Find the value of $\int_0^{\pi/2} \sin x\, dx$.

8. State the fundamental theorem of integral calculus.

9. Write down the reduction formula for $\int \sin^n x\, dx$.

10. Find the value of $\int_0^{\pi/2} \cos^5 x\, dx$.

11. If $I_n = \int_0^{\pi/2} \sin^n x\, dx$ and $J_n = \int_0^{\pi/2} \cos^n x\, dx$, then show that $I_n = J_n$.

12. Write down the value of $\int_0^{\pi/2} \cos^n x\, dx$ if n is odd.

13. What do you mean by rectification of plane curve?

14. What is the length of the curve $y = f(x)$ from $x = a$ to $x = b$?

15. Write down the parametric equation of the ellipse

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

16. What is the parametric equation of the parabola $y^2 = 4ax$?

17. Write the surface area of the solid generated by revolving the semicircular arc of radius a about the axis of x.

18. What is the volume of the sphere generated by the rotation of the circle $x^2 + y^2 = 16a^2$?

19. Write down the volume of the cone of radius of the base r and height h.

20. What is the surface area of the solid generated by the curve $y = f(x)$, intercepted between $x = a$ and $x = b$ and the axis of revolution about x-axis?

SECTION—B

Answer any *five* from the following : 2×5=10

21. Evaluate $\int \sin^{-1} x \, dx$.

22. Evaluate $\int \log x \, dx$.

23. Prove that $\int_0^a f(x)\,dx = \int_0^a f(a-x)\,dx$.

24. Show that $\int_0^{\pi/2} \frac{\sin x}{\sin x + \cos x}\,dx = \frac{\pi}{4}$.

25. If $I_n = \int x^n \cos bx \, dx$ and $J_n = \int x^n \sin bx \, dx$, then show that $bI_n = x^n \sin bx - nJ_{n-1}$.

26. If $I_n = \int_0^{\pi/4} \tan^n x \, dx$, then show that $I_{n+1} - I_{n-1} = \frac{1}{n}$.

27. Find the length of the line $y = 2x + 1$ extended from $x = 1$ to $x = 3$.

28. Show that the perimeter of the curve $x = \frac{1-t^2}{1+t^2}$, $y = \frac{2t}{1+t^2}$ is 2π.

29. Find the surface area generated by revolving the circle $x = a\cos\theta$, $y = a\sin\theta$ about its base.

30. Find the volume generated by revolving about x-axis, the area bounded by the line $y = 2x$ between $x = 0$ and $x = 1$.

SECTION—C

Answer any *five* from the following : 5×5=25

31. Find $\int \frac{x^2 \tan^{-1} x^3}{1+x^6}\,dx$.

32. Evaluate $\int \frac{(x^2+1)}{(1+x)^2} e^x \, dx$.

33. Evaluate $\int_0^1 \cot^{-1}(1 - x + x^2)\,dx$.

34. Evaluate

$$\lim_{n\to\infty}\left[\frac{n}{n^2+1^2} + \frac{n}{n^2+2^2} + \ldots + \frac{1}{2n}\right]$$

35. Find the reduction formula for $\int (\log x)^n \, dx$.

36. If $I_{m,n} = \int \sin^m x \cos nx \, dx$, then show that

$$I_{m,n} = \frac{m\cos x \cos mx + n \sin x \sin nx}{n^2 - m^2} \sin^{m-1} x - \frac{m(m-1)}{n^2 - m^2} I_{m-2,n}$$

37. Find the length of the arc of the parabola $y^2 = 4ax$ measured from the vertex to one extremity of the latus rectum.

38. Show that the length of an arc of the cycloid $x = a(t - \sin t)$, $y = a(1 - \cos t)$ is $8a$.

39. Find the surface of the solid formed by revolving the curve $r = a(1 + \cos\theta)$ about the initial line.

40. Find the volume of solid obtained by rotating about x-axis, the area of the parabola $y^2 = 4ax$ cut off by its latus rectum.

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