2021/TDC/CBCS/ODD/
MATHCC–502T/330

TDC (CBCS) Odd Semester Exam., 2021
held in March, 2022

MATHEMATICS

(5th Semester)

Course No. : MATHCC–502T

(Multivariate Calculus)

Full Marks : 70
Pass Marks : 28

Time : 3 hours

The figures in the margin indicate full marks for the questions

SECTION—A

Answer any *ten* of the following questions : 2×10=20

1. Check if the limit exists

$$\underset{(x,\, y)\to(0,\, 0)}{\text{Lt}} \frac{x^2+xy}{2xy-y^2}$$

2. Check the continuity of f at origin, where

$$f(x, y) = \begin{cases} \dfrac{xy}{\sqrt{x^2 + y^2}}, & (x, y) \neq (0, 0) \\ 1, & (x, y) = (0, 0) \end{cases}$$

3. Evaluate f_x and f_y if

$$f(x, y) = x^3 y + \sin(xy^2)$$

4. Define extreme value of a function of two variables. Give an example.

5. Find the stationary points of

$$f(x, y) = y^2 + 4xy + 3x^2 + x^3$$

6. Can you mention an extreme value of the following function?

$$f(x, y) = |x| + |y|, \ (x, y) \in \mathbb{R}^2$$

Justify your answer.

7. For the function

$$\vec{F} = yz^2 \hat{i} + xy\hat{j} + yz\hat{k}$$

verify that div curl $\vec{F} = 0$.

8. Evaluate

$$\iint (x + y)\, dx\, dy$$

over the area bounded by the lines $y = x$, $x = 3$ in the first quadrant.

9. Sketch the region of integration for the integral

$$\int_0^{\pi} \int_0^{\sin x} y\, dy\, dx$$

10. Compute the Jacobian of transformation from Cartesian to spherical polar coordinates.

11. Change the order of the integration

$$\int_0^1 dx \int_x^{\sqrt{x}} f(x, y)\, dy$$

12. Evaluate the line integral

$$\int_C xy\, dx$$

where C is the arc of the parabola $x = y^2$ from $(1, -1)$ to $(1, 1)$.

13. State Green's theorem in $\mathbb{R}^2$.

14. Using Green's theorem, deduce the expression for area of a domain bounded by a contour C regular with respect to both the axes.

15. State Gauss' divergence theorem.

SECTION—B

Answer any *five* of the following questions : 10×5=50

16. *(a)* Show that if f and g are two functions defined on some neighbourhood of (a, b) such that

$$\operatorname*{Lt}_{(x,\ y)\to(a,\ b)} f(x, y) = l \text{ and } \operatorname*{Lt}_{(x,\ y)\to(a,\ b)} g(x, y) = m$$

then

$$\operatorname*{Lt}_{(x,\ y)\to(a,\ b)} (f+g)(x, y) = l + m$$ 5

(b) Show that the function f is continuous, where

$$f(x, y) = \begin{cases} \dfrac{xy(x^2 - y^2)}{x^2 + y^2}, & (x, y) \neq (0, 0) \\ 0, & (x, y) = (0, 0) \end{cases}$$ 5

17. *(a)* Show that the function

$$f(x, y) = \begin{cases} \dfrac{xy}{\sqrt{x^2 + y^2}}, & (x, y) \neq (0, 0) \\ 0, & (x, y) = (0, 0) \end{cases}$$

possesses both the partial derivatives at (0, 0) but is not differentiable at (0, 0). 5

(b) Define directional derivative of a function of two variables at a point (a, b) in the direction of unit vector $\hat{u}$. Derive the directional derivative of

$$f(x, y) = x^2 + y^2$$

at (a, b) in the direction of unit vector $\hat{u} = u_1\hat{i} + u_2\hat{j}$. 2+3=5

18. *(a)* Investigate the function

$$f(x, y) = 2x^4 - 3x^2y + y^2$$

for extreme values. 5

(b) Find the shortest distance from the origin to the hyperbola

$$x^2 + 8xy + 7y^2 = 225, \ z = 0$$ 5

19. *(a)* If $2x + 3y + 4z = a$, show that the maximum value of $x^2y^3z^4$ is $\left(\frac{a}{9}\right)^9$. 5

(b) Find an extreme value of the function

$$f(x, y) = x^2 + 3xy + y^2 + x^3 + y^3$$ 5

20. (a) Evaluate

$$\iint_R \frac{dx\,dy}{(x+y+1)^2}$$

over the rectangle $R = [0, 1; 0, 1]$. 5

(b) Evaluate

$$\iint_R (x^2 + y^2)\,dx\,dy$$

over the region R bounded by the parabolas $y = x^2$ and $y^2 = x$. 5

21. (a) Evaluate

$$\iint_C x^3 y^2\,dx\,dy$$

where C is the circular disc $x^2 + y^2 \le a^2$. 5

(b) Using double integration, show that the area of a circle of radius r is πr^2. 5

22. (a) Compute the integral

$$\iiint_E xyz\,dx\,dy\,dz$$

where E is bounded by $x = 0$, $y = 0$, $z = 0$, $x + y + z = 1$. 5

(b) Compute the volume of the ellipsoid

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$$ 5

23. (a) Find the line integral

$$\int_C (x-y)^3\,dx + (x-y)^3\,dy$$

where C is the circle $x^2 + y^2 = a^2$ in counterclockwise direction. 5

(b) Evaluate

$$\iint_R f(x, y)\,dy\,dx$$

over the rectangle $R = [0, 1; 0, 1]$, where

$$f(x, y) = \begin{cases} x + y, & x^2 < y < 2x^2 \\ 0, & \text{elsewhere} \end{cases}$$ 5

24. (a) Compute the line integral

$$\int_C (1 - x^2)y\,dx + (1 + y^2)x\,dy$$

where C is $x^2 + y^2 = a^2$, using Green's theorem. 5

(b) Evaluate the surface integral

$$\iint_S x\,dy\,dz + dz\,dx + xz^2\,dx\,dy$$

where S is the outer part of the sphere $x^2 + y^2 + z^2 = 1$ in the first octant. 5

25. (a) Apply Stokes' theorem to evaluate

$$\int_C y\,dx + z\,dy + x\,dz$$

where C is the curve

$$x^2 + y^2 + z^2 - 2ax - 2ay = 0,\ x + y = 2a$$ 5

(b) Use Gauss' divergent theorem to evaluate

$$\iint_S y^2 z\,dx\,dy + xz\,dy\,dz + x^2 y\,dz\,dx$$

where S is the outer side of the surface in first octant formed by the paraboloid of revolution $z = x^2 + y^2$, cylinder $x^2 + y^2 = 1$ and the coordinate planes. 5

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