2021/TDC/CBCS/ODD/
MATHCC–101T/324

TDC (CBCS) Odd Semester Exam., 2021
held in March, 2022

MATHEMATICS

(1st Semester)

Course No. : MATHCC–101T

(Calculus)

Full Marks : 50
Pass Marks : 20

Time : 3 hours

The figures in the margin indicate full marks for the questions

SECTION—A

Answer any *ten* of the following questions : 2×10=20

1. If $y = \log x$, then find y_{99}.

2. Find by Leibnitz's formula, the nth derivative of $y = x^3 \sin x$.

3. If $\log y = \tan^{-1} x$, then prove that

$$(1+x^2)y_2 + (2x-1)y_1 = 0$$

4. State L'Hospital's rule.

5. Define rectangular and oblique asymptotes.

6. Find the vertical and horizontal asymptotes of the graph of

$$y = \frac{3x+1}{x^2-4}$$

7. Write the reduction formula for

$$\int_0^{\pi/2} \cos^n x\,dx$$

8. Evaluate :

$$\int_0^{\pi/2} \sin^6 x\,dx$$

9. Obtain the reduction formula for

$$\int \tan^n x\,dx$$

10. Write the Cartesian and parametric equation of a circle with radius r and centre at (a, b).

11. Let $x = \phi(t)$, $y = \psi(t)$ be the parametric equations of the curve AB, t being the parameter. Write down the formula for the arc length of AB.

12. Find the volume generated by revolving about OX, the area bounded by $y = x^3$ between $x = 0$ and $x = 2$.

13. Show that $(\vec{a} \times \vec{b}) \times \vec{c} = \vec{a} \times (\vec{b} \times \vec{c})$ iff $\vec{a}$ and $\vec{c}$ are parallel.

14. Prove that

$$[\vec{a}+\vec{b} \quad \vec{b}+\vec{c} \quad \vec{c}+\vec{a}] = 2[\vec{a} \quad \vec{b} \quad \vec{c}]$$

15. If $\vec{u}(t)$ and $\vec{v}(t)$ be two differential functions of the scalar t, then show that

$$\frac{d(\vec{u} \times \vec{v})}{dt} = \vec{u} \times \frac{d\vec{v}}{dt} + \frac{d\vec{u}}{dt} \times \vec{v}$$

SECTION—B

Answer any *ten* of the following questions : 3×10=30

16. If $y = x^{2n}$, where n is positive integer, then show that—

(a) $y_n = 2^n \{1.3.5.\cdots(2n-1)\} x^n$;

(b) $y_n = \frac{\lfloor 2n}{\lfloor n} x^n$.

17. Let

$$P_n = \frac{d^n}{dx^n}(x^n \log x)$$

Prove the relation

$$P_n = n \cdot P_{n-1} + \lfloor n-1$$

Hence show that

$$P_n = n!\left(\log x + 1 + \frac{1}{2} + \frac{1}{3} + \cdots + \frac{1}{n}\right)$$

18. If $y = \sin(m \sin^{-1} x)$, then prove that

$$(1-x^2)y_{n+2} - (2n+1)xy_{n+1} - (n^2 - m^2)y_n = 0$$

Also find $y_n(0)$.

19. Evaluate :

$$\lim_{x \to 0} \frac{xe^x - \log(1+x)}{x^2}$$

20. Find the asymptote of the curve

$$x^3 + y^3 = 3axy$$

21. Find the point of inflection, if any, of the curve

$$y = \frac{x^3}{a^2 + x^2}$$

22. Find the value of

$$\int_0^{\pi/4} \tan^6 x \, dx$$

23. Obtain the reduction formula for

$$\int \sec^n x \, dx$$

n being a positive integer greater than 1.

24. Obtain the reduction formula for

$$\int \sin^m x \cos^n x \, dx$$

where m, n are positive integers > 1.

25. Find the whole length of the curve

$$x^{2/3} + y^{2/3} = a^{2/3}$$

26. Find the length of one complete arc of the cycloid $x = a(\theta - \sin\theta)$ and $y = a(1 - \cos\theta)$.

27. Find the area of the surface of the solid generated by revolving the arc of the parabola $y^2 = 4ax$ bounded by the latus rectum about x-axis.

28. Prove that

$$(\vec{a} \times \vec{b}) \times \vec{c} = (\vec{a} \cdot \vec{c})\vec{b} - (\vec{b} \cdot \vec{c})\vec{a}$$

29. Find the parametric and non-parametric equations of the plane passing through three non-collinear points whose position vectors are $\vec{a}$, $\vec{b}$ and $\vec{c}$.

30. Show that the vector equation of the sphere on the join of two given points $\vec{a}$ and $\vec{b}$ as diameter is

$$(\vec{r}-\vec{a})\cdot(\vec{r}-\vec{b})=0$$

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