Pass 2021

2021/TDC/CBCS/ODD/
MATDSE–502T(A/B)/332

TDC (CBCS) Odd Semester Exam., 2021
held in March, 2022

MATHEMATICS

(5th Semester)

Course No. : MATDSE–502T

Full Marks : 70
Pass Marks : 28

Time : 3 hours

The figures in the margin indicate full marks for the questions

Candidates have to answer *either* Option—A *or* Option—B

OPTION—A

Course No. : MATDSE–502T (A)

(Analytical Geometry)

SECTION—A

Answer any *twenty* of the following questions as directed : 1×20=20

1. Under what condition the equation $ax^2 + 2hxy + by^2 = 0$ will represent a pair of parallel straight lines?

2. What will be the two invariants when the expression $ax^2 + 2hxy + by^2$ changes to $a'x'^2 + 2h'x'y' + b'y'^2$ by an orthogonal transformation?

3. Write the transformed equation of $x^2 - y^2 = 0$ when the origin is shifted to $(-1, 2)$.

4. Write the equations of the lines represented by $x^2 - 5xy - 6y^2 = 0$.

5. Write down the equations to the bisectors of the angles between the lines represented by the equation $x^2 + 2y^2 + 4xy = 0$.

6. Define radical axis in case of two circles.

7. State the condition under which the line $y = mx + c$ touches the circle $x^2 + y^2 = a^2$.

8. Find the point of contact if $y = mx + \frac{a}{m}$ be a tangent to the parabola $y^2 = 4ax$.

9. Under what condition the circles $x^2 + y^2 + 2g_1x + 2f_1y + c_1 = 0$ and $x^2 + y^2 + 2g_2x + 2f_2y + c_2 = 0$ cut each other orthogonally?

10. Define radical centre of three circles.

11. Write the polar equation of a conic.

12. Define pole and polar for a given curve.

13. What is the equation of polar of the point (α, β) w.r.t. the parabola $y^2 = 4ax$?

14. Find the nature of the conic $\frac{8}{r} = 4 - 5\cos\theta$.

15. Find the polar equation of the parabola whose latus rectum is 8.

16. Write the equation of a plane in normal form.

17. If two lines are coplanar, what is the shortest distance between the lines?

18. What are the centre and radius of the sphere $x^2 + y^2 + z^2 + 2x - 4y + 2z - 3 = 0$?

19. Write down the equation of the sphere with (x_1, y_1, z_1) and (x_2, y_2, z_2) as the end points of a diameter.

20. Define great circle.

21. Define a right circular cone.

22. What should be the values of l, m, n if the generators are parallel to X-axis?

23. Every _____ degree of homogeneous equation in x, y, z represents a cone with vertex at the _____. (Fill in the blanks)

24. Define cylinder.

25. What do you mean by right circular cylinder?

SECTION—B

Answer any *five* of the following questions : 2×5=10

26. Find the value of k for which the equation $2x^2+3xy-2y^2+7x+y+k=0$ represents a pair of straight lines.

27. If the equation

$$ax^2+3xy-2y^2-5x+5y+c=0$$

represents two straight lines perpendicular to each other, then find a and c.

28. Find the radical axis of the two circles $x^2+y^2+2x+4y-7=0$ and $x^2+y^2-6x+2y-5=0$

29. Show that the line $x+2y-4=0$ touches the ellipse $3x^2+4y^2=12$.

30. Find the equation of the polar of the point (2, 3) w.r.t. the circle $x^2+y^2-2x-4y+1=0$.

31. Find the point on the conic $\frac{15}{r}=1-4\cos\theta$ whose radius vector is 5.

32. Prove that the straight lines $\frac{x-1}{2}=\frac{y+1}{-3}=\frac{z+10}{8}$ and $\frac{x-4}{1}=\frac{y+3}{-4}=\frac{z+1}{7}$ intersect each other.

33. Find the equation of the sphere passing through the origin and makes intercepts a, b, c on the coordinate axes.

34. Find the equation of the cone whose vertex is origin and guiding curve is $x^2+y^2+z^2=1, 2x+3y+4z=7$.

35. What do you mean by guiding curve and generator of a cylinder?

SECTION—C

Answer any *five* of the following questions : 8×5=40

36. (a) If

$$ax^2 + 2hxy + by^2 = 1 \text{ and } a'x^2 + 2h'xy + b'y^2 = 1$$

represent the same conic, axes being rectangular, then show that $(a-b)^2 + 4h^2 = (a'-b')^2 + 4h'^2$. 4

(b) Find the equation of the pair lines through origin and perpendicular to the pair $ax^2 + 2hxy + by^2 = 0$. 4

37. (a) Show that the two straight lines through the origin which make angles 45° with the line $lx + my + n = 0$ are given by

$$(l^2 - m^2)(x^2 - y^2) + 4lmxy = 0$$ 4

(b) If two pairs of lines $x^2 - 2pxy - y^2 = 0$ and $x^2 - 2qxy - y^2 = 0$ be such that each pair bisects the angles between the other pair, then prove that $pq + 1 = 0$. 4

38. (a) Find the equation of the circle which passes through the origin and cuts orthogonally each of the circles

$$x^2 + y^2 - 6x + 8 = 0 \text{ and } x^2 + y^2 - 2x - 2y = 7$$ 4

(b) Prove that the locus of the point of intersection of two tangents to an ellipse at right angles to one another is a circle. 4

39. (a) Find the radical centre of the circles

$$x^2 + y^2 + x + 2y + 3 = 0,$$

$$x^2 + y^2 + 2x + 4y + 5 = 0 \text{ and}$$

$$x^2 + y^2 - 7x - 8y - 9 = 0$$ 3

(b) Prove that two tangents can be drawn from a point to a parabola and if these two tangents be perpendicular to each other, the locus of their point of intersection is the directrix. 5

40. (a) Find the equation of the polar of the point (x_1, y_1) w.r.t. the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$. 4

(b) If PSP' and QSQ' be two perpendicular focal chords of a conic $\frac{l}{r} = 1 + e\cos\theta$, then show that $\frac{1}{PP'} + \frac{1}{QQ'}$ = constant. 4

41. *(a)* Find the pole of the straight line $2x - y = 6$ w.r.t. the circle $5x^2 + 5y^2 = 9$. 4

(b) Find the condition that the line $\frac{l}{r} = A\cos\theta + B\sin\theta$ may be a tangent to the conic $\frac{l}{r} = 1 + e\cos\theta$. 4

42. *(a)* Find the length and the equations of the shortest distance between the lines

$$\frac{x-1}{2} = \frac{y-2}{3} = \frac{z-3}{4} \text{ and } \frac{x-2}{3} = \frac{y-4}{4} = \frac{z-5}{5}$$ 4

(b) A sphere of constant radius k passes through the origin and meets the axes in *A*, *B*, *C*. Prove that the centroid of the triangle *ABC* lies on the sphere $9(x^2 + y^2 + z^2) = 4k^2$. 4

43. *(a)* Prove that the shortest distance between the y-axis and the line

$$ax + by + cz + d = 0 = a'x + b'y + c'z + d'$$

is

$$\frac{bd' - b'd}{\sqrt{(ba' - b'a)^2 + (bc' - b'c)^2}}$$ 4

(b) Show that $2x - 6y + 3z - 49 = 0$ is a tangent plane to the sphere $x^2 + y^2 + z^2 = 9$. Find the point of contact. 4

44. *(a)* Prove that the equation of the right circular cone, whose vertex is the origin, axis is *OX* and semi-vertical angle α, is $y^2 + z^2 = x^2 \tan^2 \alpha$. 4

(b) Find the equation of the cylinder generated by the line parallel to Z-axis and passing through the curve of intersection of the plane $3x + 2y - z = 4$ and the surface $5x^2 - 2y^2 + 3z^2 = 1$. 4

45. *(a)* A variable plane is parallel to the given plane $\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 0$ and meets the axes in *A*, *B* and *C* respectively. Prove that the circle *ABC* lies on the cone

$$yz\left(\frac{b}{c} + \frac{c}{b}\right) + zx\left(\frac{a}{c} + \frac{c}{a}\right) + xy\left(\frac{a}{b} + \frac{b}{a}\right) = 0$$ 4

(b) Find the equation of the right circular cylinder whose axis is the line

$$\frac{x}{2} = \frac{y}{3} = \frac{z}{6}$$

and radius is 5 units. 4

OPTION—B

Course No. : MATDSE–502T (B)

(Probability and Statistics)

SECTION—A

Answer any *twenty* of the following questions : 1×20=20

1. Define sample space.

2. Write the sample space of the experiment of throwing a die and tossing a coin simultaneously.

3. Two dice are thrown simultaneously. What is the probability that the numbers that appear are both prime?

4. Define random variable.

5. If $E(X) = 7, E(Y) = 8$, what is $E(X+Y)$?

6. What are Bernoulli trials?

7. Write the p.m.f. of binomial distribution.

8. What is the mean of a Poisson distribution?

9. What is the p.d.f. of exponential distribution?

10. If $X \sim N(1, 2)$, what is σ?

11. What is joint probability mass function?

12. What is marginal probability function?

13. What are independent random variables?

14. What is the expectation of a linear combination of random variables?

15. Define conditional expectation.

16. Define Karl Pearson coefficient of correlation between two random variables.

17. What is regression analysis?

18. What is line of regression?

19. How is covariance between two random variables defined?

20. What is the regression line of Y on X?

21. State Chebyshev's inequality.

22. State weak law of large numbers.

23. State central limit theorem.

24. State Markov's inequality.

25. Define convergence in probability of a sequence of random variables.

SECTION—B

Answer any *five* of the following questions : 2×5=10

26. A bag contains 3 black, 2 yellow and 4 green balls. Two balls are drawn at random. What is the probability that none of them is yellow?

27. Two cards are drawn from a pack of 52 cards. What is the probability that both of them are queens?

28. Compute the mean of a binomial distribution.

29. Write some uses of normal distribution.

30. The joint probability density function of (X, Y) is given by

$$f(x, y) = \begin{cases} \frac{8}{9}xy & , 1 \le x \le y \le 2 \\ 0 & , \text{elsewhere} \end{cases}$$

Find the marginal density function of X.

31. If X is a continuous random variable with $X \ge 0$, then show that $E(X) \ge 0$.

32. Show that the correlation coefficient lies between −1 and 1.

33. If $X, Y \sim BVN(\mu_1, \mu_2, \sigma_1^2, \sigma_2^2, \rho)$ and $\rho = 0$, then show that X and Y are independent.

34. A symmetric die is thrown 600 times. Find a lower bound for the probability of getting 80 to 120 sixes.

35. State Chebyshev's theorem and prove it using Chebyshev's inequality.

SECTION—C

Answer any *five* of the following questions : 8×5=40

36. (a) If X and Y are random variables, then show that $E(X+Y) = E(X) + E(Y)$ and $E(XY) = E(X)E(Y)$. 5

(b) Let X be the number of heads in tossing three unbiased coins. Write the probability distribution of X. 3

37. *(a)* A random variable X has the following probability mass function :

X	0	1	2	3	4	5	6
$P(X)$	K	$3K$	$5K$	$7K$	$9K$	$11K$	$13K$

(i) Find the value of K.

(ii) Find $P(3 < X \leq 5)$.

(iii) Obtain the distribution function of X.

(iv) What is the smallest value of x for which $P(X \leq x) > 0 \cdot 5$? 1+1+1+1=4

(b) Let X be a random variable having p.d.f. given by

$$f(x) = \begin{cases} cx & , \quad 0 \leq c \leq 1 \\ c & , \quad 1 < x < 2 \\ -cx + 3c & , \quad 2 \leq x \leq 3 \\ 0 & , \quad \text{elsewhere} \end{cases}$$

Find c and the distribution function of X. 4

38. *(a)* Derive the probability function of Poisson distribution. 5

(b) Show that for a negative binomial distribution, the mean is less than variance. 3

(Continued)

39. *(a)* Show that mean = median for a normal distribution. 4

(b) Derive the moment-generating function of an exponential distribution and hence find the mean and variance. 4

40. *(a)* Show that the joint probability function of a two-dimensional random variable is monotonic non-decreasing. 4

(b) If x and y are random variables, then show that $E(X+Y) = E(X) + E(Y)$ provided all the expectations exist. 4

41. *(a)* Let x and y be jointly distributed with density function

$$f(x, y) = \begin{cases} 6(1 - x - y) & , \quad x > 0, y > 0, x + y < 1 \\ 0 & , \quad \text{elsewhere} \end{cases}$$

Find the marginal distributions of x and y, hence examine if x and y are independent. 5

(b) Show that the expected value of X is equal to the expectation of the conditional expectation of X given Y, i.e.,

$$E(X) = E(E(X|Y))$$ 3

(Turn Over)

42. *(a)* Show that the correlation coefficient is independent of change of origin and scale. 4

(b) Derive the equation of line of regression of Y on X. 4

43. Derive the expression for moment-generating function of a bivariate normal distribution. 8

44. *(a)* Prove Chebyshev's inequality. 4

(b) State and prove Lindeberg-Levy theorem. 4

45. *(a)* Prove the weak law of large numbers. 4

(b) State and prove Bernoulli's law of large numbers. 4

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