2023/FYUG/ODD/SEM/
STADSC–102T/074

FYUG Odd Semester Exam., 2023

(Held in 2024)

STATISTICS

(1st Semester)

Course No. : STADSC–102T

(Calculus)

Full Marks : 70
Pass Marks : 28

Time : 3 hours

The figures in the margin indicate full marks for the questions

SECTION—A

Answer *ten* questions, selecting *two* from each Unit : $2 \times 10 = 20$

UNIT—I

1. Define function and continuity of a function at a point.

2. State Leibniz's rule for successive differentiation.

3. Show that

$$\lim_{x\to 0} \frac{x \cdot e^{1/x}}{e^{1/x}+1} = 0$$

UNIT—II

4. Define Jacobian.

5. Define points of inflecion of a function. Write the criteria to find such points.

6. If $f(x, y)$ is a function of x and y, then under what condition $f(x, y)$ is neither maximum nor minimum?

UNIT—III

7. Evaluate $\int \frac{dx}{2\sqrt{x}}$.

8. Evaluate $\int_0^\infty x \cdot e^{-x}\, dx$.

9. Define beta and gamma functions.

UNIT—IV

10. Define order and degree of a differential equation with example.

11. Find the differential equation of the family of curves $y = e^{mx}$, where m is an arbitrary constant.

12. Define exact differential equation.

UNIT—V

13. Define partial differential equation with example.

14. Define order and degree of partial differential equation with example.

15. Eliminate arbitrary constants a and b from

$$Z = (x-a)^2 + (y-b)^2$$

to form the partial differential equation.

SECTION—B

Answer *five* questions, selecting *one* from each Unit : 10×5=50

UNIT—I

16. *(a)* Prove that a differentiable function is always continuous. 3

(b) If $y = \sin^{-1} x$, then show that

$$(1-x^2)y_{n+2} - (2n+1)xy_{n+1} - n^2 y_n = 0$$ 3

(c) Find

$$\lim_{x \to 0} \frac{x \cdot e^x - \log(1+x)}{x^2}$$ 4

17. (a) State and prove Euler's theorem for homogeneous function. 3

(b) Test the continuity and differentiability of the function

$$f(x) = |x - a|$$

at $x = a$. 4

(c) If $u = \log \frac{x^2 + y^2}{xy}$, then prove that

$$x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = 1$$ 3

UNIT—II

18. (a) Find the maximum and minimum value of the function

$$f(x) = 2x^3 - 21x^2 + 36x - 20$$ 3

(b) If

$$x_1 + x_2 + x_3 = u$$
$$x_2 + x_3 = uv$$
$$x_3 = uvw$$

then find

$$\frac{\partial(x_1, x_2, x_3)}{\partial(u, v, w)}$$ 4

(c) Find the point of inflection of the curve

$$y = \frac{x^3}{a^2 + x^2}$$ 3

19. (a) Examine if $x^{1/x}$ possess a maximum or minimum. 3

(b) Show that the function

$$f(x) = x^2 - 2xy + y^2 + x^2 + y^2$$

is minimum at origin. 3

(c) If

$$y_1 = \cos x_1$$
$$y_2 = \sin x_1 \cos x_2$$
$$y_3 = \sin x_1 \sin x_2 \cos x_3$$
$$\vdots \quad \vdots \quad \vdots$$
$$y_n = \sin x_1 \sin x_2 \ldots \sin x_{n-1} \cos x_n$$

then find the Jacobian of $y_1, y_2, \ldots, y_n$ with respect to $x_1, x_2, \ldots, x_n$. 4

UNIT—III

20. *(a)* Evaluate

$$\iint_E y\,dx\,dy$$

over the part of the plane bounded by the lines $y = x$ and parabola $y = 4x - x^2$. 4

(b) Evaluate

$$\iint_R \sqrt{x^2 + y^2}\,dx\,dy$$

where R is the region of XY-plane bounded by the circles $x^2 + y^2 = 4$ and $x^2 + y^2 = 9$. 6

21. *(a)* Prove that $B(m, n) = \dfrac{\Gamma(m)\Gamma(n)}{\Gamma(m+n)}$. 4

(b) Show that $\Gamma(n) = (n-1)!$. 2

(c) Prove that

$$\int_0^1 \frac{x^{m-1}(1-x)^{n-1}}{(a+bx)^{m+n}}\,dx = \frac{1}{(a+b)^m \cdot a^n} B(m, n)$$ 4

UNIT—IV

22. *(a)* Solve : 5

$$(x^2y - 2xy^2)\,dx - (x^3 - 3x^2y)\,dy = 0$$

(b) Solve : 5

$$(x^3 + 3xy^2)dx + (y^3 + 3x^2y)dy = 0$$

23. (a) Solve : 6

$$\frac{dy}{dx} + \frac{y}{(1-x^2)^{3/2}} = \frac{x + (1-x^2)^{1/2}}{(1-x^2)^2}$$

(b) Solve : 4

$$\frac{dy}{dx} = e^{x+y} + x^2 e^{x^3+y}$$

UNIT—V

24. (a) Solve : 5

$$p + 3q = 5z + \tan(y - 3x)$$

(b) Solve : 5

$$xyp + y^2q = zxy - 2x^2$$

25. (a) Solve : 6

$$\frac{d^2y}{dx^2} + 2\frac{dy}{dx} + 10y + 37\sin 3x = 0$$

(b) Solve : 4

$$\frac{d^3y}{dx^3} + 6\frac{d^2y}{dx^3} + 11\frac{dy}{dx} + 6y = 0$$

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