

2023/FYUG/ODD/SEM/ MATDSM-101T/142

FYUG Odd Semester Exam., 2023 (Held in 2024)

MATHEMATICS

(1st Semester)

Course No. : MATDSM-101T

(Calculus)

Full Marks: 70
Pass Marks: 28

Time: 3 hours

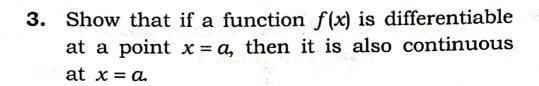
The figures in the margin indicate full marks for the questions

SECTION-A

Answer ten questions, selecting any two from each
Unit: 2×10=20

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- 1. State Cauchy's criterion for the existence of limit of a function.
- 2. Show that $\lim_{x\to 0}\cos\left(\frac{1}{x}\right)$ does not exist.



UNIT-II

MATHUMATIC

- 4. Give the geometrical meaning of Rolle's theorem.
- 5. In the mean value theorem

$$f(x+h) = f(x) + hf'(x+\theta h)$$

If $f(x) = Ax^2 + Bx + C$, where $A \neq 0$, show that

$$\theta = \frac{1}{2}$$

6. Evaluate:

$$\lim_{x \to 1} \left(\frac{x}{x - 1} - \frac{1}{\log x} \right)$$

UNIT-III

- **7.** Define homogeneous function of degree n of two variables. Is the function $(\sqrt{x} + \sqrt{y} + \sqrt{z})$ is homogeneous? If so find its degree.
- 8. If $f(x, y) = e^{x^2 + xy + y^2}$, find f_{xx} and f_{xy} .

9. If
$$u = f\left(\frac{y}{x}\right)$$
, show that

$$x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = 0$$

UNIT-IV

- 10. If f(x) is an even function, show that $\int_{-a}^{a} f(x) dx = 2 \int_{0}^{a} f(x) dx$
- 11. If f(x) = x [x], then evaluate $\int_{-1}^{1} f(x) dx$

where [x] is the integral part of x.

12. Evaluate:

$$\int_0^{\pi/2} \sin^8 x \cos^6 x \, dx$$

UNIT-V

13. Find the length of arc of the parabola $y^2 = 16x$ measured from vertex to an extremity of the latus rectum.

- 14. What do you mean by rectification of plane curve? Write the formula to find the length of the curve y = f(x) from x = a to x = b.
- **15.** Find the surface area of a solid generated by revolving the semicircular arc of radius c about the axis of x.

SECTION—B

16. If f(x) is an even function, show that

Answer five questions, selecting one from each Unit:

UNIT—I

16. (a) Using ε - δ definition of limit, evaluate

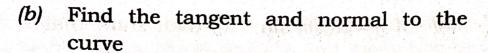
$$\lim_{x \to 0} x^2 \cos\left(\frac{1}{x}\right) \qquad 5$$

(b) Examine the differentiability of the function

alcohomo
$$f(x) = \begin{cases} x^2 \sin\left(\frac{1}{x}\right), & \text{if } x \neq 0 \text{ and in } 1 \text{ is } 1 \text{ and } 1 \text{ in }$$

at
$$x = 0$$
.

7~			
17.	· (a)	If $y \sqsubseteq_{\mathbf{c};\mathbf{w}}$	
	(1 –	If $y = \sin(m \sin^{-1} x)$, then show that $(x^2)y_{n+2} - (2n+1)xy$	
	(2	$(x^{-})y_{n+2} - (2n+1)xy$	
	\ Ein	$(m \sin^2 x)$, then show that $(m \sin^2 x)y_{n+2} - (2n+1)xy_{n+1} - (n^2 - m^2)y_n = 0$ Also find $y_n(0)$. State and prove Leit.	
	(b)	State and prove Leibnitz's theorem on successive differentiation.	5
		successive disc. Leibnitz's theorem	Ā
			2
		UNIT—II	5
18	(01)		S.
	• (a)	State and prove Lagrange's mean value theorem.	
		theorem.	
	(b)	Evaluate:	5
			5
			J
	3.4	$\lim_{x \to \infty} \left(\frac{\tan x}{x} \right)^{1/x}$	3
	0.4	$\lim_{x \to 0} \left(\frac{\tan x}{x} \right)^{1/x}$	3
19	• (a)	$\lim_{x \to 0} \left(\frac{\tan x}{x} \right)^{1/x}$	3
19	· (a)	$\lim_{x\to 0} \left(\frac{\tan x}{x}\right)^{1/x}$ Show that the maximum	3
19	• (a)	$\lim_{x\to 0} \left(\frac{\tan x}{x}\right)^{1/x}$ Show that the maximum value of $x^{1/x}$ is $e^{1/e}$.	5
19	• (a) (b)	Show that the maximum value of $x^{1/x}$ is $e^{1/e}$. Write the statement of Maximum in	
19	• (a)	Show that the maximum value of $x^{1/x}$ is $e^{1/e}$. Write the statement of Maclaurin's theorem. Also expand sin x	5
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19	• (a) (b)	Show that the maximum value of $x^{1/x}$ is $e^{1/e}$. Write the statement of Maclaurin's theorem. Also expand $\sin x$ using Maclaurin's infinite expansion. $1+4=$	5
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	• (a)	Show that the maximum value of $x^{1/x}$ is $e^{1/e}$. Write the statement of Maclaurin's theorem. Also expand $\sin x$ using Maclaurin's infinite expansion. 1+4= UNIT—III State and prove Euler's theorem on homogeneous function of degree n in	5



$$y(x-2)(x-3)-x+7=0$$

at the point where it cuts the x-axis. 5

21. (a) If

$$V = \log\left(\frac{x^3 + y^3}{x^2 + y^2}\right)$$

show that

$$x\frac{\partial V}{\partial x} + y\frac{\partial V}{\partial y} = 1$$

(b) Show that at any point on the curve $x^{m+n} = k^{m-n}y^{2n}$

the *m*th power of the subtangent varies as the *n*th power of the subnormal.

UNIT-IV

22. (a) Obtain the reduction formula for $\int \sin^m x \cos^n x \, dx$

where m, n are positive integers > 1. 5

(b) Evaluate: $\int_0^{\pi/2} \left(\frac{1}{1+\cot x}\right) dx$

(Continued)



(7)

23. (a) Obtain the reduction formula for $\int \sec^n x \, dx$

n being a positive integer greater than 1. 5

(b) Prove that

$$\int_0^{\pi/2} \log(\sin x) \, dx = -\frac{\pi}{2} \log 2$$

UNIT-V

24. (a) Find the area of the astroid $x^{2/3} + y^{2/3} = a^{2/3}$ 5

(b) Find the area bounded by the parabola $y^2 = 4ax$ and its latus rectum.

25. (a) Find the surface area of the solid generated by revolving the cycloic $x = a(\theta + \sin \theta), y = a(1 + \cos \theta)$ about its base.

(b) The circle $x^2 + y^2 = a^2$ revolves around the axis. Find the surface area and the volume of the whole surface generated. 5

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