OPTION—B

Course No. : MTMDSE–602T (B)

(Theory of Equations)

SECTION—A

nswer any *twenty* of the following as directed : 1×20=20

1. What is the remainder when $3x^2+4x-11$ is divided by $x-1$?

2. State fundamental theorem of algebra.

3. What will be the nature of the roots if the signs of the terms of an equation be all positive?

4. State remainder theorem.

5. If $f(\alpha)$ and $f(\beta)$ be of opposite signs, then what can you say about the number of real roots between α and β of $f(x)=0$?

5. Find the sum and product of the roots of the equation $4x^3+7x-3=0$.

. If α and β are the roots of $x^2-2x+3=0$, then find the equation whose roots are $\frac{1}{\alpha}, \frac{1}{\beta}$.

8. If one root of $5x^2 + 13x + k = 0$ is reciprocal of the other, then find the value of k.

9. If α, β, γ be the roots of the cubic equation $x^3 + px^2 + qx + r = 0$, then find the value of $\Sigma\alpha^2$.

10. If the sum of the roots of the equation $\lambda x^2 + 2x + 3\lambda = 0$ be equal to their product, then find the value of λ.

11. Name any one method to solve a cubic equation.

12. Write down the standard form of a biquadratic equation.

13. Under what transformation the equation $ax^3 + 3bx^2 + 3cx + d = 0$ reduces to $Z^3 + 3HZ + G = 0$?

14. If α, β, γ be the roots of the equation $x^3 + px^2 + qx + r = 0$, then find the value of $\Sigma\frac{1}{\alpha}$.

15. If α, β, γ and δ be the roots of the biquadratic equation $x^4 + px^3 + qx^2 + rx + s = 0$, then find the value of $\Sigma\alpha\beta$.

16. If all the roots of $f(x) = ax^3 + bx^2 + cx + d$ are real, then find the number of real roots of $f'(x)$.

17. Define superior limit of roots.

18. Find the number of imaginary roots of $x^5 + x^4 + x^2 - 25x - 36 = 0$.

19. Write the condition that the roots of the cubic equation $x^3 + 3Hx + G = 0$ should be real.

20. Let $f(x) = x^3 - 2x - 5$, find its first derived function $f_1(x)$.

21. Whether the equation $x^4 - 4x^3 + 8x + 4 = 0$ has commensurable roots?

22. Find the condition that the roots of the equation $ax^2 + 2bx + c = 0$ are real and unequal.

23. An equation in which the coefficient of the first term is unity, and the coefficients of the other terms are whole numbers, cannot have a commensurable root which is not a whole number.

(Write True or False)

24. Write the conditions that the roots of the cubic equation $Z^3 + 3HZ + G = 0$ are all real and unequal.

25. Horner's method is applied in solving any numerical equation to find both the commensurable and incommensurable roots.

(Write True or False)

SECTION—B

Answer any *five* of the following questions : $2 \times 5 = 10$

26. Find the quotient and remainder when $x^4 + 5x^3 + 4x^2 + 8x - 20$ is divided by $x - 1$.

27. Find the equation whose roots are $2, -3, 4, -1$.

28. If $2 + i\sqrt{3}$ is a root of the equation $x^2 + px + q = 0$, where p and q are real, then find (p, q).

29. If the difference of the roots of $x^2 - px + 8 = 0$ be 2, then find the value of p.

30. If α, β, γ be the roots of the equation $x^3 + px^2 + qx + r = 0$, then find the value of $(\alpha + \beta)(\beta + \gamma)(\gamma + \alpha)$.

31. State Newton's theorem on the sums of powers of roots.

32. Find all the roots of the equation $x^4 - 2x^3 - 19x^2 + 68x - 60 = 0$ which lie between -6 and 6.

33. Find an approximate value of the positive root of the equation $x^3 - 2x - 5 = 0$.

34. Find the integral roots of the equation $x^4 - 2x^3 - 13x^2 + 38x - 24 = 0$.

35. Find all the commensurable roots of $2x^3 - 31x^2 + 112x + 64 = 0$.

SECTION—C

Answer any *five* of the following questions : 8×5=40

36. *(a)* Express $3x^3 - 4x^2 + 5x + 6$ as a polynomial in $x + 1$. 4

(b) Prove that the equation $x^3 + x^2 - 5x - 1 = 0$ has one positive root lying in (1, 2) and two negative roots lying in (−1, 0) and (−3, −2). 4

37. *(a)* Apply Descarte's rule of signs to find the nature of the roots of the equation $x^4 + qx^2 + rx - s = 0$ (q, r, s being positive). 4

(b) Solve the equation

$x^4 - 3x^3 - 5x^2 + 9x - 2 = 0$, $(2 - \sqrt{3})$

being one of its roots. 4

38. *(a)* If α, β, γ be the roots of the biquadratic equation $x^4 + px^3 + qx^2 + rx + s = 0$, then find the value of $\Sigma\alpha^4$. 4

(b) If α, β, γ be the roots of the equation $x^3 + 2x^2 + 1 = 0$, then find the equation whose roots are $\alpha + \frac{1}{\alpha}, \beta + \frac{1}{\beta}, \gamma + \frac{1}{\gamma}$. 4

39. (a) If $\alpha_1, \alpha_2, \cdots, \alpha_n$ be the roots of the equation

$$x^n + \rho_1 x^{n-1} + \cdots + \rho_{n-1} x + \rho_n = 0, \rho_n \neq 0,$$

Find the value of $\Sigma \frac{\alpha_1^2 + \alpha_2^2}{\alpha_1 \alpha_2}$. 4

(b) Find the equation whose roots are the cubes of the roots of the equation $x^4 - 2x^3 + x^2 + 3x - 1 = 0$. 4

40. (a) Solve $x^3 - 18x - 35 = 0$ by Cardan's method. 4

(b) Solve the equation $x^4 - 2x^2 + 8x - 3 = 0$. 4

41. (a) Reduce the equation

$$x^3 + 6x^2 - 12x + 32 = 0$$

to its standard form and then solve the equation. 5

(b) If α, β, γ be the roots of the equation $x^3 + px + q = 0$, then find the value of $\Sigma \frac{1}{\alpha + \beta}$. 3

(a) Find the number and position of the real roots of the equation $x^4 - 2x^3 - 7x^2 + 10x + 10 = 0$. 4

(b) Apply Sturm's theorem to analyze the equation $x^4 - 4x^3 + 7x^2 - 6x - 4 = 0$. 4

3. (a) Calculate Sturm's functions for the following equation and show that four roots are imaginary : 4

$$3x^5 + 5x^3 + 2 = 0$$

(b) Prove that the roots of the equation $x^3 - (a^2 + b^2 + c^2)x - 2abc = 0$ are all real and solve it when two of the quantities become equal. 4

4. (a) Find the positive root of the equation $x^3 + x^2 + x - 100 = 0$ correct to four decimal places. 4

(b) Find by Horner's method, the real positive root of the equation $8x^3 - 10x^2 - 3x - 7 = 0$ which lies between 1 and 2. 4

45. (a) Find a root of the equation $x^3 - 2x - 5 = 0$ correct to two places of decimal by Newton's method of approximation. 4

(b) Find in the form of a continued fraction the positive root of the equation $x^3 - 2x - 5 = 0$. 4

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