2019/TDC/ODD/SEM/CSCHCC–102T/181

TDC (CBCS) Odd Semester Exam., 2019

COMPUTER SCIENCE

(1st Semester)

Course No. : CSCHCC–102T

(Discrete Structures)

Full Marks : 70
Pass Marks : 28

Time : 3 hours

The figures in the margin indicate full marks for the questions

UNIT—I

1. Answer any *two* questions from the following :   2×2=4

   (a) Define injective function with an example.

   (b) What do you mean by equivalence relation?

   (c) What is transitive relation? Give an example.

     (Turn Over)

2. (a) (i) Show that $f : R \rightarrow R - \{1\}$ given by $f(x) = (x+1)/(x-1)$ is onto. 5

   (ii) If A, B and C are any sets, then show that the following identities hold true : 5

   $$A \cap (B \times C) \equiv (A \cap B) \times (A \cap C)$$

   $$A - (B \cap C) \equiv (A - B) \cup (A - C)$$

   Or

   (b) (i) Let R and S be the relations on $A = \{1, 2, 3\}$ and

   $$R = \{(1, 1), (1, 2), (2, 3), (3, 1), (3, 3)\}$$

   $$S = \{(1, 2), (1, 3), (2, 1), (3, 3)\}$$

   Find $R \circ S$ and $S \circ S$. 5

   (ii) Define reflexive, symmetric and antisymmetric relations with examples of each. 5

Unit—II

3. Answer any *two* questions from the following : 2×2=4

   (a) Explain θ notation with graphical representation.

   (b) Show that $\sum\limits_{k=0}^{n} 3^k = O(3^n)$.

   (c) Differentiate the uses of ω and big O notation.

4. (a) (i) Prove that

   $$f(n) = 4n^3 + 5n^2 + 7n + 3 = O(n^3)$$ 3

   (ii) Show that $2x^3 + x^2 - 3x + 2 = \Omega(x^3)$. 3

   (iii) Determine whether $\log n!$ is $\theta(n \log n)$. Justify your answer. Also show that $n \log n$ is $O(\log n!)$. 4

   Or

   (b) Discuss about different bounding summation methods. 10

Unit—III

5. Answer any *two* from the following questions : 2×2=4

   (a) Find the first three terms of the recurrence relation

   $$a_n = a_{n-1} + a_{n-2}$$

   for all integers $n \geq 2$, $a_0 = 1$, $a_1 = 1$.

   (b) Write master theorem.

   (c) Find the recurrence relation of the sequence $S = \{1, 5, 18, 29, 61, \cdots\}$.

6. (a) (i) Find the first six terms of the following recurrence relations : 3+3=6

$$a_n = na_{n-1} + a_{n-2}^2, \quad a_0 = -1, \quad a_1 = 0$$

and $a_n = a_{n-1} - a_{n-2} + a_{n-3}$, $a_0 = 1$, $a_1 = 1$, $a_2 = 2$.

   (ii) Solve the recurrence relation $a_n - 4a_{n-2} = 0$ for $n \geq 2$ with $a_0 = 1$ and $a_1 = 1$. 4

Or

(b) (i) Solve the recurrence relation $a_n = (n+1)a_{n-1}$ with initial condition $a_0 = 2$. 3

   (ii) Find the solution to the recurrence relation

$$a_n = 6a_{n-1} - 11a_{n-2} + 6a_{n-3}$$

with the initial condition $a_0 = 2$, $a_1 = 5$ and $a_2 = 15$. 4

   (iii) Find the generating function for the finite sequence 1, 1, 1, 1, 1, 1. 3

UNIT—IV

7. Answer any *two* questions from the following : 2×2=4

(a) Define graph.

(b) What do you mean by spanning tree? Explain with an example.
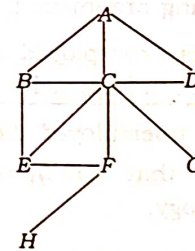
(c) What is isomorphic graph? Give an example.

8. (a) (i) Prove that the number of vertices of odd degree in a graph is always even. 4

   (ii) Describe Eulerian and Hamiltonian graph. 6

Or

(b) (i) Write down the algorithm for depth-first search (DFS). Draw the DFS spanning tree of the following graph : 6

   (ii) What is tree? Write the properties of a tree. 4

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UNIT—V

9. Answer any *two* from the following questions : 2×2=4

(a) Write down the truth table for 'conjunction' and 'disjunction'.

(b) Define well-formed formula.

(c) Translate the following sentence into propositional form :

"If it is not raining and I have the time, then I will go to a movie."

10. (a) (i) Show that

$(P \Rightarrow Q) \wedge (R \Rightarrow Q) \equiv (P \vee R) \Rightarrow Q$ 4

(ii) Determine the validity of the following argument : 4

If a man is unemployed, he is unhappy.
If a man is unhappy, he dies young.

Therefore, unemployed dies young.

(iii) Show that $(p \wedge q) \Rightarrow (p \vee q)$ is a tautology. 2

Or

(b) (i) Prove that the following argument is valid : 3

$p \rightarrow 7q, \ r \rightarrow q, \ r \vdash 7p$

(ii) Construct the truth table for

$\alpha = (P \vee Q) \Rightarrow ((P \vee R) \Rightarrow (R \vee Q))$ 3

(iii) Discuss the validity of the following argument : 4

All educated persons are well-behaved.

Ram is educated.
No well-behaved person is quarrelsome.

Therefore, Ram is not quarrelsome.

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