2018/TDC/ODD/CSCC–102T/031

TDC (CBCS) Odd Semester Exam., 2018

COMPUTER SCIENCE

(1st Semester)

Course No. : CSCHCC–102T

(Discrete Structures)

Full Marks : 70

Pass Marks : 28

Time : 3 hours

The figures in the margin indicate full marks for the questions

STA : To answer **two** questions out of **three** from each Unit @ *2* marks

Broad Type : To answer **one** question out of **two** from each Unit @ *10* marks

UNIT—I

1. Answer any *two* from the following : 2×2=4

(a) Determine the power set of

$$A = \{a, b, c, d\}$$

(b) Consider the following relation in A :

$$R = \{(1, 1), (2, 2), (2, 3), (3, 2), (4, 2), (4, 4)\}$$

(i) Is R reflexive? Give reasons.

(ii) Is R symmetric? Give reasons.

(c) If A, B, C be any three sets, prove that

$$A \times (B \cap C) = (A \times B) \cap (A \times C)$$

2. Answer any *one* from the following :

(a) (i) In a class of 25 students, 12 students in all have taken Economics; 8 have taken Economics but not Politics. Find the number of students who have taken Economics and Politics. 4

(ii) Define bijective function. Show that the function $f: Q \to Q$ defined by $f(x) = 3x + 2$ is a bijective function, where Q is the set of rational numbers. 1+2=3

(iii) Let $f: R \to R$ be a function defined as $f(x) = x^3 - 1$ and $g: R \to R$ be another function defined as $g(x) = 2x^2 - 2$. What will be the functions $f \circ g$ and $g \circ f$? 3

(b) (i) Suppose that the relations R_1 and R_2 on a set A are represented by the matrices

$$M_{R_1} = \begin{bmatrix} 1 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix} \text{ and } M_{R_2} = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 1 & 0 & 0 \end{bmatrix}$$

What are the matrices representing $R_1 \cup R_2$ and $R_1 \cap R_2$? 3

(ii) What is partial ordering relation? Explain with example. 3

(iii) Define pigeonhole principle. Prove that if n pigeons are assigned to m pigeonholes, then at least one pigeonhole contains two or more pigeons $(m < n)$. 1+3=4

UNIT—II

3. Answer any *two* from the following : 2×2=4

(a) Is $2^{n+1} = O(2^n)$?

(b) Is $2^{2n} = O(2^n)$?

(c) Show that $x^2 + x + 1$ is $\theta(x^2)$.

4. Answer any *one* from the following : 10

(a) Define Big 'O', theta and omega notations as used in the analysis of algorithm. Bring out the differences among them using graphical representation. Give examples for each notation. 10

(b) (i) Use the definition of big 'O' to prove that

$$f(n) = 5n^4 - 37n^3 + 13n - 4 = O(n^4)$$ 3

(ii) Let $f(x) = (x+1)\log(x^2+1)$. Estimate the growth of $f(x)$. 4

(iii) Find out whether the following equalities are correct : 3

$$10n^2 + 4n + 2 = \Omega(n^2)$$

UNIT—III

5. Answer any *two* from the following : 2×2=4

(a) Find the recurrence relation of the sequence

$$S = \{5, 8, 11, 14, 17, \ldots\}$$

(b) Find the recurrence relation of the Fibonacci sequence of numbers

$$S = \{1, 1, 2, 3, 5, 8, 13, \ldots\}$$

(c) Find the first four terms of the recurrence relation

$$a_k = a_{k-1} + 3a_{k-2}$$

for all integers $k \geq 2$, $a_0 = 1$, $a_1 = 2$.

6. Answer any *one* from the following :

(a) (i) Define recurrence relation. Explain with example. 2+2=4

(ii) Discuss Tower of Hanoi problem. Determine the recurrence relation of the Tower of Hanoi problem. 4

(iii) Find the 6 terms of following recurrence relation : 2

$$a_k = k \cdot (a_{k-1})^2$$

for all integers $k \geq 1$, $a_0 = 1$

(b) (i) Solve the recurrence relation 4

$$a_n = a_{n-1} + 2, \ n \geq 2$$

(ii) Solve the recurrence relation

$$a_n = 4(a_{n-1} - a_{n-2})$$

with initial conditions $a_0 = a_1 = 1$. 3

(iii) Find the generating function of a sequence $\{a_k\}$, if $a_k = 2 + 3k$. 3

UNIT—IV

7. Answer any *two* from the following : 2×2=4

(a) Define complete and regular graphs with examples.

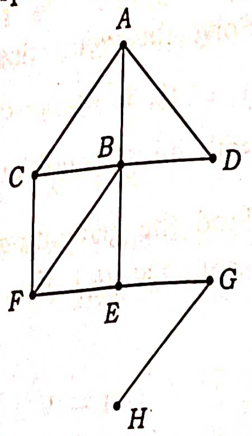
(b) Define adjacency matrix and incidence matrix in a graph with suitable examples.

(c) Write any algorithm for finding minimal spanning tree.

8. Answer any *one* from the following :

(a) (i) Prove that a finite connected graph G is Eulerian if and only if each vertex has even degree. 5

(ii) Write down the breadth first search (BFS) algorithm. Draw the BFS spanning tree of the following : 5



(b) (i) Show that the sum of in-degrees of all the nodes of a simple digraph is equal to the sum of out-degrees of all its nodes and that this sum is equal to the number of edges of the graph. 4

(ii) Explain graph coloring with example. Also write the Welch-Powell algorithm. 4+2=6

UNIT—V

9. Answer any *two* from the following : 2×2=4

(a) Explain, with example, a proposition.

(b) Construct the truth table for $(P \vee Q) \vee \rceil P$.

(c) Prove that $(P \to Q) \Leftrightarrow (\rceil P \vee Q)$.

10. Answer any *one* from the following :

(a) (i) Show that

$$P \to (Q \to R) \Leftrightarrow P \to (\rceil Q \vee R) \Leftrightarrow (P \wedge Q) \to R$$ 3

(ii) Test the validity of the following argument : 4

If I study, then I will not fail in Maths.

If I do not play basketball, then I will study.

But I failed in Maths.

Therefore, I must have played basketball.

(iii) Define tautology, contradiction and contingency. Give example. 3

(b) (i) Write the negation of each of the following statements : 3

Statement 1 : He swims if and only if the water is warm.

Statement 2 : If it snows, then they do not drive the car.

(ii) Define universal quantifier and existential quantifier. Give examples. 3

(iii) Prove that the following arguments is valid : 4

$$P \to \rceil Q, \quad R \to Q, \quad R \vdash \rceil P$$

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